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An optimal algorithm for solving the dynamic lot-sizing model with learning and forgetting in setups and production

Huan Neng Chiu[a], Hsin Min Chen[b,*]

[a] Department of Industrial Management, National Taiwan University of Science and Technology, 43, Keelung Road, Section 4, Taipei 106, Taiwan
[b] Department of Business Management, National United University, No.1, Lien-Kung, Kung-Ching Li, Miaoli 360, Taiwan

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Abstract

This paper studies the problem of incorporating both learning and forgetting in setups and production into the dynamic lot-sizing model to obtain an optimal production policy, including the optimal number of production runs and the optimal production quantities during the finite period planning horizon. Since the unit production cost is variable due to the effects of learning and forgetting, the first-in-first-out (FIFO) inventory costing method is used in our model. After deriving the relevant cost functions, we develop the multi-dimensional forward dynamic programming (MDFDP) algorithm based on two important properties that can be proved to be able to reduce the computational complexity. A numerical example is illustrated and solved using our refined MDFDP algorithm. The results from our computational experiment show that the optimal number of production runs decreases with the increase of the learning or forgetting rates, while the optimal total cost increases with the increase of one of the above four rates. Production learning has the greatest influence on the optimal total cost among the four parameters. The interactive effects of five demand patterns and nine relationships generated by the four rates on the optimal number of production runs and the optimal total cost are also examined.

Keywords: Learning and forgetting effects; Dynamic lot-sizing model; FIFO inventory costing method

1. Introduction

Owing to the increasing emphasis on time-based competition, the importance of learning and forgetting effects on manufacturing has been widely recognized. Both effects on the continuous review system with a constant demand rate have been studied by Keachie and Fontana (1966), Spradlin and Pierce (1967), Adler and Nanda (1974), Carlson (1975), Sule (1978, 1981), Axsäter and Elmaghraby (1981), Elmaghraby (1990), and Jaber and Bonney (1997a, 1998, 2001). The above studies only considered learning and forgetting effects on production. Another study conducted by Li and Cheng (1994) was more general in that the economic production quantity (EPQ) model involved learning in setups and both learning and forgetting in production. Jaber and Bonney (1999) surveyed the above models and suggested possible extensions to the

*Corresponding author. Tel.: +886-37-381590; fax: 886-37-332396.
E-mail address: hmchen@nuu.edu.tw (H.M. Chen).

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lot size problem in which both learning and forgetting are incorporated into both setups and production. They also suggested that their earlier work may be extended to the model of Wagner and Whitin by including a finite planning horizon with zero inventories at the beginning of the initial cycle and the end of the last cycle. However, few papers have incorporated both effects into the dynamic lot-sizing problems with discrete time-varying demand. Chand and Sethi (1990) considered the dynamic lot-sizing problem in a pure setup learning environment in which only setup costs were susceptible to improvements. They developed a forward dynamic programming algorithm, which can be used on a rolling horizon basis, for infinite horizon problems. Tzur (1996) extended Chand and Sethi's work to a more general model, which allows the total setup cost to be a general nondecreasing (but not necessarily concave) function of the number of setups. Recently, Chiu (1997) incorporated learning and forgetting effects on production into the dynamic lot-sizing model. Furthermore, he also extended the optimal Wagner and Whitin (1958) algorithm and three existing heuristic models.

Unlike previous works, this paper studies the problem of incorporating both learning and forgetting in setups and production into the dynamic lot-sizing model to obtain an optimal production policy, including the number of production runs, lot sizes, and time points for starting setups and production. Since the period-demand and finite periods of the planning horizon are assumed in this paper, but setups and production times are scheduled continuously, the proposed model is virtually a mix of discrete and continuous models. As far as we know, few papers have studied this model.

The setup time and unit production time are assumed to have learning phenomena, and are represented as power functions of the cumulative number of repetitions. The forgetting effect is mainly caused by a break between two consecutive production runs and leads to retrogression in learning. Besides the quantity produced to date and the length of the interruption, other factors such as the availability of the same personnel, tooling, and methods that have a direct effect on the degree of human forgetting were also considered in Anderlohr (1969) and Cochran (1973). Globerson et al. (1989) showed that the degree of forgetting is a function of the break length and the level of experience gained prior to the break in a laboratory experiment. In fact, a variety of factors influence the forgetting effect like the break length, previous experience, job complexity, the work engaged in during the interruption period, the cycle time of the task, the relearning curve, and a single relearning observation (e.g., repair or maintenance) (Dar-El, 2000, pp. 83–92). Jaber and Bonney (1996) proposed a mathematical model in which the forgetting slope is dependent on three factors (i.e., the equivalent accumulated output of continuous production by the point of interruption, the minimum break under total forgetting, and the learning slope). They (Jaber and Bonney, 1997b) also compared their model with two existing models. Their model is more realistic, and their predicted time was very close to the experimental data provided by Globerson et al. (1989). For simplicity, we assumed fixed forgetting rates in setups and production, as adopted by Li and Cheng (1994), to make our proposed multi-dimensional forward dynamic programming (MDFDP) algorithm more tractable. Since the production cost of each unit is not identical due to learning and forgetting, the FIFO inventory costing method is used in this paper.

In the next section, the notations used throughout this paper are defined, and basic assumptions are given. Section 3 then presents a general description of the model and formulates relevant cost functions for each production run. Subsequently, Section 4 develops the refined MDFDP algorithm by applying two important properties, and an example is also provided. An experiment conducted to analyze the effects of relevant parameters on the optimal solution is discussed in Section 5. Finally, Section 6 concludes the paper with a brief summary of the results.

2. Notations and assumptions

The following notations will be used throughout this study:

Parameters

N	length of the planning horizon expressed in periods
d_i	demand in a given period i, $d_1 > 0$ and $d_i \geqslant 0$
r_s	fixed learning rate in setups, $0 < r_s \leqslant 1$
b_s	learning coefficient associated with setups, $b_s = -\log r_s / \log 2$
f_s	fixed forgetting rate in setups, $0 \leqslant f_s \leqslant 1$
r_p	fixed learning rate in production, $0 < r_p \leqslant 1$
b_p	learning coefficient associated with production, $b_p = -\log r_p / \log 2$
f_p	fixed forgetting rate in production, $0 \leqslant f_p \leqslant 1$
θ	fixed production capacity per period (in man-periods)
C_o	direct labor cost per man-period
C_m	direct material cost and overhead per unit
C_h	fixed carrying cost rate per period

Decision variables

n	total number of production runs planned for the entire planning horizon
q_j	number of units produced in the jth production run.

Intermediate variables

i	period count that denotes the time interval between the time points of $i-1$ and i, $i = 1, 2, \ldots, N$
j	production run count, $j = 1, 2, \ldots,$ and $j \leqslant N$
$D(i,m)$	cumulative units of demand from a specific period i to period m. That is, $D(i,m) = d_i + d_{i+1} + \cdots + d_{m-1} + d_m = \sum_{\alpha=1}^{m} d\alpha$ and $i \leqslant m \leqslant N$
$I(i)$	inventory at the end of period i after the demand d_i is satisfied, $I(i) \geqslant 0$
Q_j	cumulative units produced from the first production run to the jth production run. That is, $Q_j = q_1 + q_2 + \cdots + q_j$ and $Q_0 = 0$
S_j	time (in man-periods) required to set up the jth production run
$t_{j,x}$	time (in man-periods) required to produce the xth cumulative unit of the jth production run, where $1 \leqslant x \leqslant q_j$
P_j	production time in the jth production run, $P_j = \sum_{x=1}^{q_j} t_{j,x}$
A_j	time point at which setup of the jth production run begins (see Fig. 1)
B_j	time point at which production in the jth production run begins
M_j	number of periods whose demand is satisfied during the production phase (Phase I) in the jth production run
K_j	number of periods whose demand is satisfied during the non-production phase (Phase II) in the jth production run
SC_j	the setup cost of the jth production run
PC_j	the production cost, including the direct labor cost, direct material cost and overhead, for the jth production run
WC_j	the inventory carrying cost incurred during the production phase in the jth production run
HC_j	the inventory carrying cost incurred during the non-production phase in the jth production run

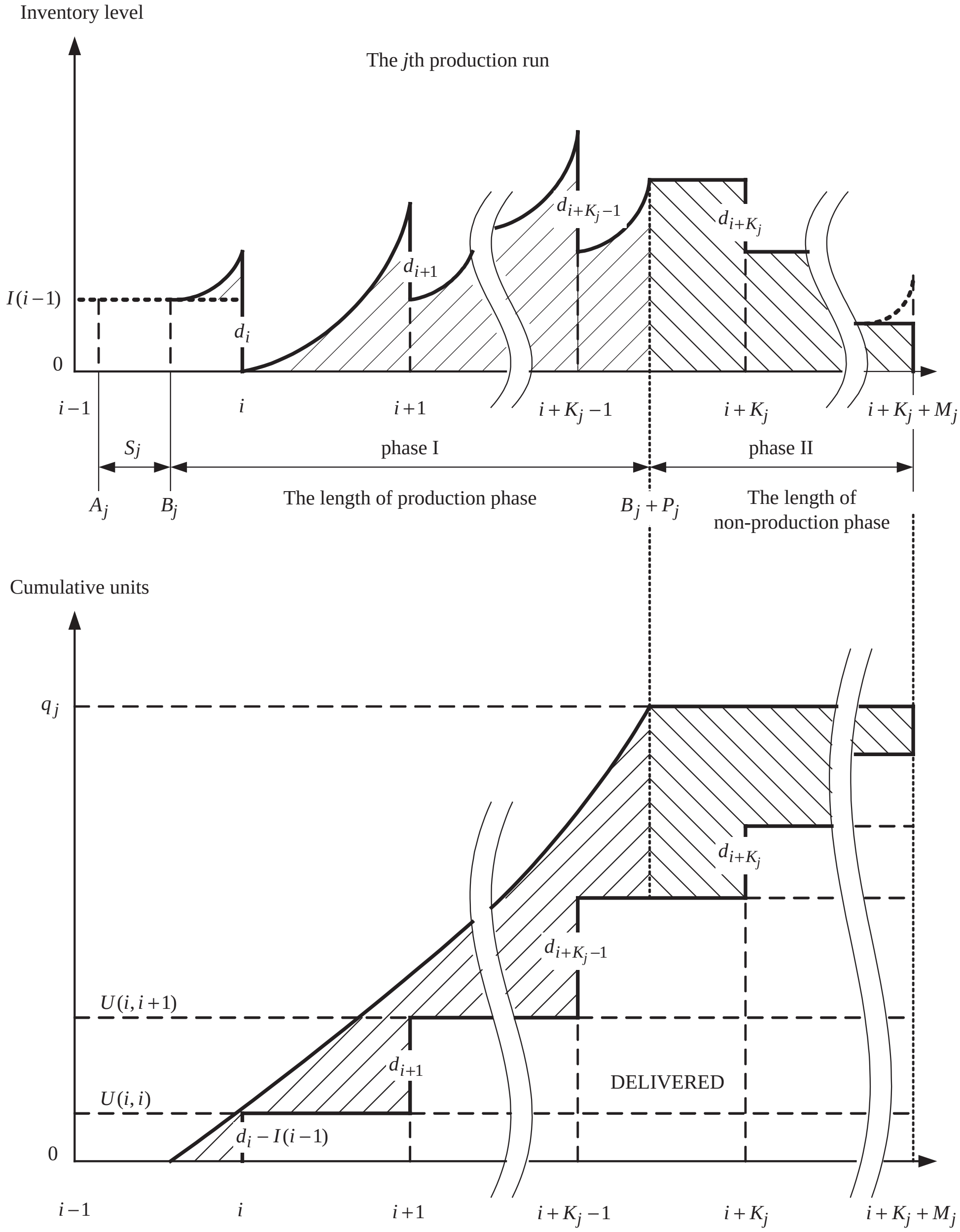

Fig. 1. Inventory levels and cumulative production units of the jth production run.

The objective of this paper is to obtain the optimal solution for the above defined decision variables that minimizes the total cost during the planning horizon i.e., minimize

$$\sum_{j=1}^{n} (SC_j + PC_j + WC_j + HC_j).$$

The following assumptions are made herein:

(1) The single-stage manufacturing system considers only one product, and the product is not subject to deterioration, obsolescence, or perishability during the finite planning horizon.
(2) The demand in the form of $d_1, d_2, \ldots, d_N$ is known but varies from one period to another. The demand for each period is scheduled to be delivered (i.e., to be satisfied) at the end of that period, and each period has the same length of time.
(3) The beginning inventory in period 1 and the ending inventory in period N both equal zero (i.e., $I(0) = I(N) = 0$). No shortages or stockouts are permitted during the planning horizon. The production capacity per period, θ, can satisfy the period demand. A mathematical expression for the production capacity constraint is $S_j + \sum_{x=1}^{d_i} t_{j,x} \leqslant \theta$ man-periods, where the jth production run during the planning horizon is performed in period i. Without loss of the generality, we assume that $\theta = 1$ in this paper.
(4) To achieve the objectives of lower inventory and on-time delivery, the start times of setup and production in a production period are delayed as long as possible without incurring shortages. Production starts immediately when setup is finished. A setup is not necessarily incurred in every production period, but only occurs after non-production (idle time).
(5) The FIFO rule is used to govern delivery units of the product produced.
(6) Both the setup time and unit production time decrease as a result of learning. A fixed fraction of the total setup learning is lost (i.e., forgotten or retrogressed) due to a manufacturing interruption between two consecutive setups. Forgetting is similarly applied to production. The two forgetting rates (i.e., f_s and f_p) have been defined previously. This forgetting assumption in production was used by Li and Cheng (1994). The time required to set up the first production run, denoted by S_1, and the time required to produce the first unit of the first production run, $t_{1,1}$, are both known.
(7) Cost parameters C_o, C_m and C_h do not change with time. The direct labor cost per period is constant since we assume that the same skilled workers perform the setup and production jobs. It is also assumed that the total overhead during the planning horizon can be estimated and allocated to the total production quantities (Q_n). Hence, the value of C_m (i.e., the sum of the unit direct material cost and unit overhead) is fixed. Similarly, this fixed value of C_h can be easily estimated based on the current cost of capital.
(8) The carrying cost for a unit of the product is proportional to its production cost and is calculated based on the time length from its completion time to the time when it is delivered.

3. Model description

The learning functions without forgetting in setups and production are

$$S_j = S_1[(j-1)+1]^{-b_s} = S_1 j^{-b_s} \tag{1}$$

and

$$t_{j,x} = t_{1,1}(Q_{j-1}+x)^{-b_p}, \tag{2}$$

where $1 \leqslant x \leqslant q_j$. From Assumption (6), the time required to set up the jth production run is

$$S_j = S_1[(1-f_s)(j-1)+1]^{-b_s}, \tag{3}$$

where $1-f_s$ represents the retentive proportion of the total learning obtained in the previous $j-1$ setups. Similarly, the production time required to produce the xth unit in the jth production run is

$$t_{j,x} = t_{1,1}[(1-f_p)Q_{j-1}+x]^{-b_p}. \tag{4}$$

Obviously, S_j in Eq. (3) equals S_1 when $j=1$ (the first production run), and $t_{j,x}$ in Eq. (4) equals $t_{1,1}$ when $j=1$ and $x=1$. In addition, Eq. (4) implies that a fraction of the total learning defined by Li and Cheng (1994, p. 121, Eq. (2)) is lost between production lots. Alternatively, if we make the remembered learning assumption under which the loss is related to the cumulative units remembered, then Eq. (4) becomes $t_{j,x} = t_{1,1}[\sum_{i=1}^{j-1}(1-f_{\rm p})^{j-i}q_i + x]^{-b_{\rm p}}$. As stated by Li and Cheng (1994), such an assumption would lead to a more complex model to which the dynamic programming approach could not be applied.

From Eq. (3) and Assumption (7), the setup cost of the jth production run is

$$SC_j = C_{\rm o}S_j = C_{\rm o}S_1[(1-f_{\rm s})(j-1)+1]^{-b_{\rm s}}. \tag{5}$$

The production cost, including the direct labor cost, direct material cost and overhead, for the jth production run can be derived from Eq. (4) and Assumption (7). The result is given by

$$\begin{aligned} PC_j &= C_{\rm o}P_j + C_{\rm m}q_j = C_{\rm o}\sum_{x=1}^{q_j} t_{j,x} + C_{\rm m}q_j \\ &= C_{\rm o}t_{1,1}\sum_{x=1}^{q_j}[(1-f_{\rm p})Q_{j-1}+x]^{-b_{\rm p}} + C_{\rm m}q_j. \end{aligned} \tag{6}$$

As shown in Fig. 1, the jth production run is supposed to start production at time B_j in period i (i.e., $i-1<B_j \geqslant 1$). Because stockouts are not allowed, as described in Assumption (3), $0<d_i - I(i-1) \leqslant q_j \leqslant D(1,N) - Q_{j-1}$. Meanwhile, the time at which the jth production run begins to produce can be determined by

$$B_j = i - \sum_{x=1}^{d_i - I(i-1)} t_{j,x} > i-1, \tag{7}$$

where i here represents the time length from the beginning of period 1 to the end of period i. The time at which setup of the jth production run begins is

$$A_j = B_j - S_j \geqslant i-1. \tag{8}$$

The carrying cost for the units produced in the jth production run can be divided into two parts. One part of the carrying cost (see the left shaded area in Fig. 1) is calculated in Phase I, while another part of the carrying cost (see the right shaded area in Fig. 1) is computed in Phase II.

In Phase I, the number of periods in which each period-demand is satisfied by the quantity produced in the jth production run is

$$K_j = \lfloor B_j + P_j \rfloor - (i-1), \tag{9}$$

where $\lfloor B_j + P_j \rfloor$ denotes the largest integer no greater than $B_j + P_j$. To simplify our presentation, we define that $U(i,w) = D(i,w) - I(i-1)$ and $i \leqslant w \leqslant N$, given $I(i-1)$. The carrying cost based on every unit production cost in this phase (i.e., the time interval between B_j and $B_j + P_j$) can then be derived as shown in Appendix A. The result is

$$\begin{aligned} WC_j = C_{\rm h}\Bigg\{ &\sum_{x=1}^{q_j-1}\left[(C_{\rm o}t_{j,x} + C_{\rm m})\left(\sum_{y=x+1}^{q_j} t_{j,y}\right)\right] - \left[C_{\rm o}\sum_{x=1}^{U(i,i)} t_{j,x} + C_{\rm m}U(i,i)\right](B_j + P_j - i) \\ &- \sum_{w=i}^{i+K_j-2}\left(C_{\rm o}\sum_{x=U(i,w)+1}^{U(i,w+1)} t_{j,x} + C_{\rm m}d_{w+1}\right)[B_j + P_j - (w+1)]\Bigg\}. \end{aligned} \tag{10}$$

Thus, the number of periods in which each period-demand is satisfied in Phase II is given by

$$M_j = \max\{\text{integer } g \mid D[1,(i-1)+K_j+g] \leqslant Q_j\}. \tag{11}$$

The carrying cost in this phase (from time $B_j + P_j$ to time $i + K_j + M_j$), calculated in Appendix B, can be presented as

$$HC_j = C_h \left\{ \sum_{\alpha=i+K_j}^{i+K_j+M_j-1} \left(C_o \sum_{x=U(i,\alpha-1)+1}^{U(i,\alpha)} t_{j,x} + C_m d_\alpha \right) [\alpha - (B_j + P_j)] + \left[C_o \sum_{x=U(i,i+K_j+M_j-1)+1}^{q_j} t_{j,x} + C_m(Q_j - D(1, i + K_j + M_j - 1)) \right] [i + K_j + M_j - (B_j + P_j)] \right\}. \tag{12}$$

Consequently, the total cost of the jth production run, which starts production in period i, is

$$\begin{aligned} TC_j &= TC(i, j, Q_{j-1}, q_j) \\ &= SC_j + PC_j + WC_j + HC_j, \end{aligned} \tag{13}$$

where SC_j, PC_j, WC_j, and HC_j are given in Eqs. (5), (6), (10), and (12), respectively.

It should be noted here that the time length of Phase II in the jth production run should be long enough so that the $(j+1)$th production run can be setup and satisfies the net demand (i.e., $d_{i+K_j+M_j} - I(i + K_j + M_j - 1)$) at the end of period $i + K_j + M_j$. That is,

$$S_{j+1} + \sum_{x=1}^{d_{i+K_j+M_j} - I(i+K_j+M_j-1)} t_{j+1,x} \leqslant i + K_j + M_j - (B_j + P_j), \tag{14}$$

where $I(i + K_j + M_j - 1) = Q_j - \sum_{\alpha=1}^{i+K_j+M_j-1} d_\alpha$.

The mathematical model of this research problem is as follows:

$$\text{Minimize} \sum_{j=1}^{n} (SC_j + PC_j + WC_j + HC_j)$$

subject to

$$S_j + \sum_{x=1}^{d_i} t_{j,x} \leqslant 1 \quad \text{for } 1 \leqslant j \leqslant i \leqslant N,$$

$$0 \leqslant I(i) \leqslant D(1, N) - D(1, i), \text{ for } i = 1, 2, \ldots, N,$$

$$d_i \leqslant I(i-1) + q_j,$$

$$0 < q_j \leqslant D(1, N) - Q_{j-1},$$

$$I(0) = 0,$$

$$1 \leqslant n \leqslant N.$$

The first inequality, $S_j + \sum_{x=1}^{d_i} t_{j,x} \leqslant 1$, expresses the capacity constraint, as described in Assumption (3). The second inequality, $0 \leqslant I(i) \leqslant D(1, N) - D(1, i)$, implies that $I(N) = 0$. The third inequality, $d_i \leqslant I(i-1) + q_j$, represents the assumption under which no shortages are permitted. The last inequality, $0 < q_j \leqslant D(1, N) - Q_{j-1}$, constrains the number of units produced in the jth production run that does not exceed an upper limit under the assumption that $I(N) = 0$. The upper limit is determined by subtracting Q_{j-1} from the total demand during the planning horizon.

4. The optimal multi-dimensional forward dynamic programming algorithm

Since the total cost of the jth production run, as shown in Eq. (13), depends on i, j, Q_{j-1}, and q_j, and the mathematical model mentioned in Section 3 cannot be solved directly, the proposed MDFDP algorithm refined by applying Properties 1 and 2 can be used to solve the optimal values of $q_1, q_2, \ldots, q_n$ and n.

Property 1. *The optimal solution does not include a production run started in a period in which the beginning inventory is large or equal to the demand of that period.*

Proof. Suppose the jth production run is performed and produces q units in period i, where $I(i-1) \geqslant d_i$ and $q \leqslant q_j$. According to Assumption (3), postponement of the jth production run to the period $i+1$ is beneficial since the savings obtained in the carrying cost is

$$C_h \left\{ C_o t_{1,1} \sum_{x=1}^{q} [(1-f_p)Q_{j-1} + x]^{-b_p} + C_m q \right\} > 0. \quad \square$$

First, let $L(j)$ be the period in which the jth production run is set up and begins to perform production. The total cost function is defined as follows:

$F[L(j), j, Q_j]$ = the minimum total cost from the first production run to the jth production run, given that the jth production run is set up and begins production in period $L(j)$, where $j \leqslant L(j) \leqslant N$, that the cumulative production quantities is Q_j, and that Q_j is sufficient to satisfy the demand from period 1 to period $L(j)$.

Second, the recurrence relation is

$$\begin{aligned} F[L(j), j, Q_j] &= \min\{TC[L(j), j, Q_j - q_j, q_j] + F[L(j-1), j-1, Q_j - q_j] \,|\, L(j-1) < L(j) \leqslant N, \\ &\quad 0 \leqslant I(L(j)) \leqslant D(1,N) - D(1,L(j)), d_{L(j)} \leqslant Q_j - D(1, L(j)-1), \text{and } q_j \leqslant D(1,N) - Q_{j-1}\}. \end{aligned} \quad (15)$$

Third, the boundary conditions are $F(0,0,0) = 0$, $F(i,0,0) \to \infty$ for $i \geqslant 1$, $F(0,j,0) \to \infty$ for $j \geqslant 1$, and $F(0,0,Q_j) \to \infty$ for $Q_j \geqslant 1$. Finally, the optimal solution is

$$F_n^* = F[L(n^*), n^*, D(1,N)] = \min\{F_n\}, \text{ where } n = 1, 2, \ldots, N$$

and

$$F_n = F[L(n), n, D(1,N)] = \min\{TC[L(n), n, D(1,N) - q_n, q_n] + F[L(n-1), n-1, D(1,N) - q_n]\}.$$

As a result, the optimal values of q_j^* for $j = 1, 2, \ldots, n$ can be obtained by using the backtracking process. Here, the computational complexity of Eq. (15) is $O(N(D(1,N))^2)$. Further, improvement of the computational efficiency can be achieved by means of the following property:

Property 2. *If the production learning rate is fixed and the unit inventory carrying cost per period for the product is proportional to the unit production cost, which is variable due to learning, then the zero inventory property holds.*

Proof. Since the unit production time decreases with the increase in the number of units produced as a result of the fixed production learning rate, both the unit production cost and the unit inventory carrying cost per period are nonincreasing (concave) functions. From Taha (1997, p. 462), it is easy to show that $I(i-1)q_j = 0$ for all i (where $I(i-1)$ is the beginning inventory in period i, and j is the count of the next production run occurring in period i). $\square$

From Property 2, Eq. (15) can be simplified to obtain

$$F(i,j,Q_j) = \min\Bigg\{ TC[i,j,D(1,i-1),q_j] + F[L(j-1),j-1,D(1,i-1)] | L(j-1) \leqslant i \leqslant N,$$
$$I(i-1)=0, d_i>0, \text{ and } q_j = \sum_{\alpha=i}^{\lambda} d_\alpha, \text{where } \lambda = i, i+1, \ldots, N \Bigg\}. \quad (16)$$

The computational complexity of the proposed MDFDP algorithm can be reduced from $O(N(D(1,N))^2)$ to $O(N^3)$ since $N \leqslant D(1,N)$.

Example. A producer of industrial vehicles carried out pilot production to satisfy orders for a new type of straddle carrier. The finished carriers were periodically delivered by train, and the production manager was confronted with the following ordering situation for the first 6 periods:

Period i	1	2	3	4	5	6
Demand d_i	6	9	11	5	3	15

Given $S_1 = 0.25$, $t_{1,1} = 0.05$, $C_o = 1{,}000$, $C_m = 500$, and $C_h = 0.05$, suppose that $r_s = 0.80$, $f_s = 0.60$, $r_p = 0.90$, and $f_p = 0.40$. Using the refined MDFDP algorithm, the best production policies for $n = 1, 2, \ldots,$ and 6 were those summarized in Table 1. As n increases, the inventory carrying cost reduces but setups and production costs increase. Therefore, the production policy with an adequate value of n is advantageous. In this example, zero inventories are encountered 8.25% (i.e., $0.4950/6 \times 100\%$) and 20.77%

Table 1
Results for the numerical example

n	j	A_j	S_j	B_j	Q_j	P_j	SC_j	PC_j	WC_j	HC_j	TC_j	F_n	F_n^*
1	1	0.4950	0.2500	0.7450	49	1.5795	250.00	26079.50	643.63	2088.44	29061.60	29061.60	
2	1	0.4950	0.2500	0.7450	31	1.0660	250.00	16566.00	265.94	683.14	17765.10		
	2	4.6810	0.2243	4.9053	18	0.5437	224.34	9543.72	84.69	218.99	10071.70	27836.80	
3	1	0.4950*	0.2500	0.7450	15	0.5692	250.00	8069.19	49.16	165.09	8533.44		
	2	2.4098	0.2243	2.6341	19	0.6121	224.34	10112.10	70.49	239.59	10646.50		
	3	5.3412	0.2069	5.5481	15	0.4519	206.90	7951.91	82.70	0.00	8241.51	27421.40	**27421.40**
4	1	0.4950	0.2500	0.7450	15	0.5692	250.00	8069.19	49.16	165.09	8533.44		
	2	2.4098	0.2243	2.6341	11	0.3659	224.34	5865.90	47.82	0.00	6138.06		
	3	3.6327	0.2069	3.8396	8	0.2538	206.90	4253.79	10.94	72.23	4543.86		
	4	5.3542	0.1939	5.5481	15	0.4519	193.96	7951.91	82.70	0.00	8228.57	27443.90	
5	1	0.4950	0.2500	0.7450	6	0.2550	250.00	3255.04	16.31	0.00	3521.34		
	2	1.4484	0.2243	1.6727	9	0.3273	224.34	4827.29	34.08	0.00	5085.71		
	3	2.4272	0.2069	2.6341	11	0.3659	206.90	5865.90	47.82	0.00	6120.62		
	4	3.6457	0.1939	3.8396	8	0.2538	193.96	4253.79	10.94	72.23	4530.91		
	5	5.3643	0.1838	5.5481	15	0.4519	183.80	7951.91	82.70	0.00	8218.42	27477.00	
6	1	0.4950	0.2500	0.7450	6	0.2550	250.00	3255.04	16.31	0.00	3521.34		
	2	1.4484	0.2243	1.6727	9	0.3273	224.34	4827.29	34.08	0.00	5085.71		
	3	2.4272	0.2069	2.6341	11	0.3659	206.90	5865.90	47.82	0.00	6120.62		
	4	3.6457	0.1939	3.8396	5	0.1604	193.96	2660.40	8.47	0.00	2862.82		
	5	4.7215	0.1838	4.9053	3	0.0947	183.80	1594.72	2.51	0.00	1781.03		
	6	5.3726	0.1755	5.5481	15	0.4519	175.53	7951.91	82.70	0.00	8210.14	27581.70	

Note: *For example, $n = 3$, the time point for starting setup for the first production run (i.e., $j = 1$) is 0.4950. The duration of the setup is 0.2500; consequently, setup ends at time 0.7450 ($=0.4950+0.2500$).

(i.e., $(0.4950 + 0.4098 + 0.3412)/6 \times 100\%$) of the time for $n = 1$ and 3, respectively. When n increases from 1 to 3, setups and production costs increase by 484.84 and the inventory carrying cost reduces by 2,125.04. Hence, the total cost reduces by 1,640.20. However, as n increases from 3 to 4, setups and production costs increase by 201.55 but the inventory carrying cost only reduces by 179.05. The total cost increases by 22.50. Similarly, the total costs are increasing as n increases from 4 to 6. As a result, the optimal production policy was based on three runs (i.e., $n = 3$) during the 6-period planning horizon (i.e., $N = 6$). The three runs were set up at times 0.4950 (see the note in Table 1), 2.4098, and 5.3412. Each production run was started immediately when the corresponding setup was finished; i.e., the three runs were started at 0.7450 (also see note in Table 1), 2.6341, and 5.5481. In fact, the three runs produced 15 units, 19 units, and 15 units in period 1, 3, and 6, respectively. The minimum total cost was 27,421.40.

5. Computational experience

We conducted an experiment to explore the effects of learning, forgetting, and the demand pattern on the total cost and the number of production runs. The proposed MDFDP algorithm was programmed in Visual C++ 6.0 and run on a PC with a Pentium III 600. A series of problems generated from $D(1, N) = 60$ were tested. For each test problem, the fixed parameters, including N, S_1, $t_{1,1}$, C_o, C_m, and C_h, were assigned the same values presented in the previous section. The various values for each of the other parameters were $r_s = 0.6$, 0.8, and 1.0; $f_s = 0.0$, 0.5, and 1.0; $r_p = 0.6$, 0.8, and 1.0; and $f_p = 0.0$, 0.5, and 1.0. In addition, the five types of demand patterns were chosen as follows:

Type I. Demand concentrated in the early and late periods: 15, 10, 5, 5, 10, 15.

Type II. Demand concentrated in the middle periods: 5, 10, 15, 15, 10, 5.

Type III. Equal demand in all periods: 10, 10, 10, 10, 10, 10.

Type IV. Gradually descending demand: 15, 15, 10, 10, 5, 5.

Type V. Gradually ascending demand: 5, 5, 10, 10, 15, 15.

A total of 405 (i.e., $3 \times 3 \times 3 \times 3 \times 5$) test problems were generated. Tables 2, 3 and 4 present the results obtained by using the proposed MDFDP algorithm. They are explained in the following:

(1) Table 2 shows that the average optimal number of production runs decrease slightly with the increase of the values of r_s, f_s, r_p, or f_p.

Table 2
Average optimal number of production runs obtained by using the proposed algorithm

Parameters		$r_p = 0.6$			$r_p = 0.8$			$r_p = 1.0$			Row average	Overall average
		$f_p = 0.0$	$f_p = 0.5$	$f_p = 1.0$	$f_p = 0.0$	$f_p = 0.5$	$f_p = 1.0$	$f_p = 0.0$	$f_p = 0.05$	$f_p = 1.0$		
$r_s = 0.6$	$f_s = 0.0$	5.00	5.00	4.00	5.00	5.00	3.77	4.15	4.15	4.15	4.47	
	$f_s = 0.5$	5.00	5.00	3.38	5.00	4.85	3.15	3.54	3.54	3.54	4.11	3.94
	$f_s = 1.0$	3.54	3.54	3.00	3.54	3.54	3.00	3.00	3.00	3.00	3.24	
$r_s = 0.8$	$f_s = 0.0$	4.93	4.93	3.93	4.93	4.93	3.67	4.00	4.00	4.00	4.37	
	$f_s = 0.5$	4.93	4.93	3.40	4.93	4.80	3.13	3.47	3.47	3.47	4.06	3.90
	$f_s = 1.0$	3.60	3.60	3.00	3.60	3.60	3.00	3.00	3.00	3.00	3.27	
$r_s = 1.0$	$f_s = 0.0$	4.93	4.93	3.93	4.93	4.93	3.67	4.00	4.00	4.00	4.37	
	$f_s = 0.5$	4.93	4.93	3.40	4.93	4.80	3.13	3.47	3.47	3.47	4.06	3.90
	$f_s = 1.0$	3.60	3.60	3.00	3.60	3.60	3.00	3.00	3.00	3.00	3.27	
Column average		4.50	4.50	3.45	4.50	4.45	3.28	3.51	3.51	3.51		
Overall average			4.15			4.08			3.51			

Table 3
Average optimal total cost obtained by using the proposed algorithm

Parameter		$r_s = 0.6$			$r_p = 0.8$			$r_p = 1.0$			Row average	Overall average
		$f_p = 0.0$	$f_p = 0.5$	$f_p = 1.0$	$f_p = 0.0$	$f_p = 0.5$	$f_p = 1.0$	$f_p = 0.0$	$f_p = 0.5$	$f_p = 1.0$		
$r_s = 0.6$	$f_s = 0.0$	31477.39	31571.51	32091.80	32310.31	32462.36	32939.05	34355.82	34355.82	34355.82	32879.99	
	$f_s = 0.5$	31577.97	31672.09	32164.17	32410.87	32562.36	32998.28	34436.78	34436.78	34436.78	32966.23	32987.81
	$f_s = 1.0$	31823.78	31897.08	32268.05	32626.41	32750.39	33087.75	34533.80	34533.80	34533.80	33117.21	
$r_s = 0.8$	$f_s = 0.0$	31500.68	31592.17	32106.30	32333.73	32482.82	32949.99	34371.25	34371.25	34371.25	32897.72	
	$f_s = 0.5$	31594.29	31685.79	32173.69	32427.33	32575.93	33004.25	34444.34	34444.34	34444.34	32977.14	32997.09
	$f_s = 1.0$	31821.20	31893.18	32269.78	32626.10	32748.86	33087.16	34533.80	34533.80	34533.80	33116.41	
$r_s = 1.0$	$f_s = 0.0$	31500.68	31592.17	32106.30	32333.73	32482.82	32949.99	34371.25	34371.25	34371.25	32897.72	
	$f_s = 0.5$	31594.29	31685.79	32173.69	32427.33	32575.93	33004.25	34444.34	34444.34	34444.34	32977.14	32997.09
	$f_s = 1.0$	31821.20	31893.18	32269.78	32626.10	32748.86	33087.16	34533.80	34533.80	34533.80	33116.41	
Column average		31634.61	31720.33	32180.40	32457.99	32598.93	33011.99	34447.24	34447.24	34447.24		
Overall average			31845.11			32689.63			34447.24			

Table 4
A summary of the average optimal number of production runs and average optimal total cost

Relationship of parameters		Average optimal number of production runs						Average optimal total cost					
		Demand pattern					Row average	Demand pattern					Row average
		I	II	III	IV	V		I	II	III	IV	V	
$r_s < r_p$	$f_s < f_p$	4.44	4.33	4.67	4.33	4.33	4.42	33766.22	33768.52	33751.28	33763.71	33770.08	33763.96
	$f_s = f_p$	4.11	4.11	4.33	4.11	4.00	4.13	33774.43	33775.39	33758.59	33773.88	33773.76	33771.21
	$f_s > f_p$	3.67	3.67	3.67	3.44	3.67	3.62	33836.53	33834.70	33834.51	33837.32	33834.44	33835.50
$r_s = r_p$	$f_s < f_p$	3.89	4.00	4.00	3.89	3.89	3.93	33031.52	33036.44	33058.59	33027.90	33036.67	33038.22
	$f_s = f_p$	4.11	4.00	4.33	4.11	4.11	4.13	32949.64	32952.58	32959.48	32941.31	32948.70	32950.34
	$f_s > f_p$	4.00	4.00	3.67	3.56	4.00	3.85	32938.48	32935.37	32970.76	32941.11	32935.12	32944.17
$r_s > r_p$	$f_s < f_p$	3.67	3.56	3.33	3.22	3.56	3.47	32358.97	32368.51	32396.21	32328.56	32363.58	32363.17
	$f_s = f_p$	3.89	3.89	3.67	3.44	3.89	3.76	32206.89	32208.32	32251.80	32190.78	32205.21	32212.60
	$f_s > f_p$	4.11	4.11	3.33	3.22	4.11	3.78	32081.36	32074.81	32154.47	32085.52	32077.00	32094.63
Column average		3.99	3.96	3.89	3.70	3.95		32993.78	32994.96	33015.08	32987.79	32993.84	

(2) Table 3 indicates that the average optimal total cost increased with the increase of the values of r_s, f_s, r_p, or f_p. It is apparent that the increase of the value of each parameter led directly to a higher setup or unit production cost. In addition, production learning had the greatest influence on total cost among the four parameters.

(3) The observation in Table 2 implies that the effects of r_p on the number of production runs are more influential than that of r_s. For instance, as r_p and r_s decrease from 1.0 to 0.6, variations in the optimal number of production runs are 18.23% (i.e., $(4.15 - 3.51)/3.51 \times 100\%$) and 1.03% (i.e., $(3.94 - 3.90)/3.90 \times 100\%$), respectively.

(4) In Table 3, it can be seen that the effect of r_p on the total cost is more significant than that of r_s on the total cost. For example, as r_p and r_s go from 1.0 to 0.6, variations of total cost are 7.55% (i.e., (34,447.24−31,845.11)/34,447.24%) and 0.03% (i.e., (32,997.09−32,987.81)/32,997.09×100%), respectively.

(5) Tables 2 and 3 also further reveal the important result that the smaller the values of r_s and r_p are, the more influential f_s and f_p are. This result is consistent with the findings of Jaber and Bonney (1996) and

Jaber and Kher (2002) in which the forgetting effects are dependent on the learning effects. In Table 2, as r_s decreases from 1.0 to 0.6, the effect of f_s on the optimal number of production runs (shown in Row average of Table 2) increases from 33.64% (i.e., $(4.37-3.27)/3.27\times 100\%$) to 37.96% (i.e., $(4.47-3.24)/3.24\times 100\%$). Similarly, as r_p decreases from 1.0 to 0.6, the effect of f_p on the optimal number of production runs (shown in Column average of Table 2) increases from 0.00% (i.e., $(3.51-3.51)/3.51\times 100\%$) to 30.88% (i.e., $(4.50-3.45)/3.45\times 100\%$). Results in Table 3 also show that effects of f_s and f_p on total cost increase as r_s and r_p decrease from 1.0 to 0.6, respectively.

(6) The optimal number of production runs and the optimal total cost were insensitive to the demand pattern, as shown in Table 4. Nine relationships among r_s, f_s, r_p, and f_p are shown in Table 4. It can be observed from the first three relationships that when $r_s < r_p$, the optimal number of production runs decreased as the forgetting rate in setups (f_s) relative to the forgetting rate in production (f_p) increased. The main reason is that the smaller r_s and the larger f_s incurred a higher cost in setups. The next three relationships led to the same results. However, the last three relationships exhibited the opposite phenomenon since the effects on production surpassed those on setups.

6. Conclusions

This study has presented an effective approach to handling the complex dynamic lot-sizing model, in which learning and forgetting in setups and production are considered simultaneously. In fact, the proposed MDFDP model is a mix of discrete and continuous ones. This inevitably causes intractability in obtaining the optimal solution, including the optimal number of production runs and the optimal production quantities during the planning horizon. Fortunately, we have developed two important properties that have been proven able to reduce the computational complexity.

The results shown in our computational experience have indicated that the average optimal number of production runs decreases as one rate increases, and that the other three rates remain fixed. The average optimal total cost increases as one of the above four rates increases. Furthermore, production learning has the greatest influence on the optimal total cost, as compared with the other three parameters. The effects of production learning on the number of production runs and total cost are more influential than that of setup learning. The results are also consistent with the important findings of previous works in which the forgetting effects are dependent on the learning effects. This paper also provides insight useful to practitioners and researchers in understanding the interactive effects of the five demand patterns and nine relationships generated by learning and forgetting rates on the average optimal number of production runs and the average optimal total cost.

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Appendix A

The derivation of the inventory carrying cost incurred in Phase I.

As shown in Fig. 1, the demand of K_j periods (from period i to period $i+K_j-1$) is satisfied in Phase I. The inventory level at the beginning of period i is $I(i-1)$. Given that $U(i,w)=(\sum_{\alpha=i}^{w} d_\alpha)-I(i-1)$, the units produced and delivered in this phase are $d_i-I(i-1)$, $d_{i+1}, \ldots,$ and d_{i+K_j-1}, respectively. Hence, the production cost for each delivery can be given, respectively, as follows:

$$C_{d_i}=C_o\sum_{x=1}^{d_i-I(i-1)} t_{j,x}+C_m[d_i-I(i-1)]=C_o\sum_{x=1}^{U(i,i)} t_{j,x}+C_m U(i,i),$$

$$C_{d_{i+1}}=C_o\sum_{x=d_i-I(i-1)+1}^{d_{i+1}+d_i-I(i-1)} t_{j,x}+C_m d_{i+1}=C_o\sum_{U(i,i)+1}^{U(i,i+1)} t_{j,x}+C_m d_{i+1},\ \ldots,$$

and

$$C_{d_{i+K_j-1}}=C_o\sum_{x=d_{i+K_j-2}+\cdots+d_i-I(i-1)+1}^{d_{i+K_j-1}+\cdots+d_i-I(i-1)} t_{j,x}+C_m d_{i+K_j-1}=C_o\sum_{x=U(i,i+K_j-2)+1}^{U(i,i+K_j-1)} t_{j,x}+C_m d_{i+K_j-1}.$$

The inventory carrying cost incurred in this phase is

$$\begin{aligned}
WC_j=C_h\Bigg\{&(C_o t_{j,1}+C_m)(t_{j,2}+t_{j,3}+\cdots+t_{j,q_j-1}+t_{j,q_j})\\
&+(C_o t_{j,2}+C_m)(t_{j,3}+t_{j,4}+\cdots+t_{j,q_j-1}+t_{j,q_j})+\cdots+(C_o t_{j,q_j-2}+C_m)(t_{j,q_j-1}+t_{j,q_j})+(C_o t_{j,q_j-1}+C_m)(t_{j,q_j})\\
&-\left[C_o\sum_{x=1}^{U(i,i)} t_{j,x}+C_m U(i,i)\right](B_j+P_j-i)-\sum_{w=i}^{i+K_j-2}\left[C_o\sum_{x=U(i,w)+1}^{U(i,w+1)} t_{j,x}+C_m d_{w+1}\right][B_j+P_j-(w+1)]\Bigg\}\\
=C_h\Bigg\{&\sum_{x=1}^{q_{j-1}}\left[(C_o t_{j,x}+C_m)\left(\sum_{y=x+1}^{q_j} t_{j,y}\right)\right]-\left[C_o\sum_{x=1}^{U(i,i)} t_{j,x}+C_m U(i,i)\right](B_j+P_j-i)\\
&-\sum_{w=i}^{i+K_j-2}\left(C_o\sum_{x=U(i,w)+1}^{U(i,w+1)} t_{j,x}+C_m d_{w+1}\right)[B_j+P_j-(w+1)]\Bigg\}. \qquad (A.1)
\end{aligned}$$

Appendix B

The derivation of the inventory carrying cost incurred in Phase II.

Given that the production of the jth production run completed in period $i+K_j$ (i.e., $i+K_j-1\leqslant B_j+P_j<i+K_j$) and the demands of M_j periods (from period $i+K_j$ to period $i+K_j+M_j-1$) are satisfied in Phase II, as shown in Fig. 1. The d_{i+K_j}, $d_{i+K_j+1}, \ldots,$ and $d_{i+K_j+M_j-1}$ units produced in the jth production run are delivered at the end of period $i+K_j$, $i+K_j+1, \ldots,$ and $i+K_j+M_j-1$, respectively. Since $U(i,w)=\left(\sum_{\alpha=i}^{w} d_\alpha\right)-I(i-1)$, the production cost for each delivery can be calculated by

$$C_{d_{i+K_j}}=C_o\sum_{x=d_{i+K_j-1}+\cdots+d_i-I(i-1)+1}^{d_{i+K_j}+\cdots+d_i-I(i-1)} t_{j,x}+C_m d_{i+K_j}=C_o\sum_{x=U(i,i+K_j-1)+1}^{U(i,i+K_j)} t_{j,x}+C_m d_{i+K_j},$$

$$C_{d_{i+K_j+1}} = C_o \sum_{x=d_{i+K_j}+\cdots+d_i-I(i-1)+1}^{d_{i+K_j+1}+\cdots+d_i-I(i-1)} t_{j,x} + C_m d_{i+K_j+1} = C_o \sum_{x=U(i,i+K_j)+1}^{U(i,i+K_j+1)} t_{j,x} + C_m d_{i+K_j+1}, \ldots,$$

and

$$\begin{aligned} C_{d_{i+K_j+M_j-1}} &= C_o \sum_{x=d_{i+K_j+M_j-2}+\cdots+d_i-I(i-1)+1}^{d_{i+K_j+M_j-1}+\cdots+d_i-I(i-1)} t_{j,x} + C_m d_{i+K_j+M_j-1} \\ &= C_o \sum_{x=U(i,i+K_j+M_j-2)+1}^{U(i,i+K_j+M_j-1)} t_{j,x} + C_m d_{i+K_j+M_j-1}. \end{aligned}$$

The production cost for the residual units, which are produced in the jth production run but left for the first delivery in the next production run, is

$$\begin{aligned} C_{d_{i+K_j+M_j}} &= C_o \sum_{x=d_{i+K_j+M_j-1}+\cdots+d_i-I(i-1)+1}^{q_j} t_{j,x} + C_m[Q_j - D(1, i+K_j+M_j-1)] \\ &= C_o \sum_{x=U(i,i+K_j+M_j-1)+1}^{q_j} t_{j,x} + C_m[Q_j - D(1, i+K_j+M_j-1)]. \end{aligned}$$

As a result, the inventory carrying cost incurred during Phase II can be expressed as

$$\begin{aligned} HC_j = C_h \Bigg\{ & \sum_{\alpha=i+K_j}^{i+K_j+M_j-1} \left(C_o \sum_{x=U(i,\alpha-1)+1}^{U(i,\alpha)} t_{j,x} + C_m d_\alpha \right) [\alpha - (B_j + P_j)] \\ & + \left[C_o \sum_{x=U(i,i+K_j+M_j-1)+1}^{q_j} t_{j,x} + C_m(Q_j - D(1, i+K_j+M_j-1)) \right] [i + K_j + M_j - (B_j + P_j)] \Bigg\}. \end{aligned} \quad \text{(B.1)}$$

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