

# Outsourcing or capacity expansions: Application of activity-based costing model on joint products decisions

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## Abstract

Because of capacity constraints, the companies who produce joint products have to assess the desirability of further processing joint products beyond the split-off point. Especially in a situation in which market demands exceed the company's production capacity. In order to satisfy customer's orders and maximize total profits, these companies must study the feasibility of expanding capacity or outsourcing the production of parts. The aim of this paper is to develop an ABC joint products decision model which incorporates capacity expansions and outsourcing features, by using a mathematical programming approach. With the model presented in this paper, we can evaluate the comparative benefits of expanding the various kinds of capacity and outsourcing simultaneously. By applying this model, the companies who produce joint products can derive an optimal decision about further processing, capacity expansions or outsourcing.

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*Keywords:* Activity-based costing (ABC); Outsourcing; Joint products

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## 1. Introduction

Many companies, such as petroleum refiners, lumber mills, meat packers or flour mills, produce a multitude of products in their manufacturing processes. Joint products are produced simultaneously by common process or series of processes. All costs incurred before the split-off point of joint products are referred to as joint costs, and costs incurred for further processing and disposal are referred to as separable costs. Due to capacity constraints, when joint products can either be sold at the split-off point or after further separate processing, the companies who produce joint products have to assess the desirability of further processing of joint products beyond the split-off point. Especially in a situation in which market demands exceed the company's production capacity. In order to satisfy customer's orders and maximize total profits, the feasible way for these companies includes capacity expansions or outsourcing part of the products. Such decisions involve resource allocation and require accurate analysis of relevant costs associated with each option.

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Efficient firms allocate their own resources to those activities within the value chain for which they enjoy a comparative advantage over competitors [1]. Other activities not enjoying such advantages are increasingly outsourced to external suppliers. Outsourcing is expected to involve production cost savings relative to internal production because outside suppliers benefit from economies of scale, smoother production schedules and centralization of expertise [2,3]. Based on a survey of more than 1200 companies, Deavers [4] identified the top five reasons for outsourcing as: to improve company focus, access to world-class capabilities, acceleration of benefits from reengineering, sharing of risk and freeing of resources for other purposes. De Kok [5] considers outsourcing as a measure to allocate capacity. The additional capacity needs are not postponed but are instead outsourced. Thus, in the situation that market demands exceed current capacity, outsourcing may be a good way to obtain the advantages of cost saving and share the risk.

In recent years, because of dissatisfaction with the distortions created by traditional costing systems [6], activity-based costing (ABC) has become a popular cost management technique both with accounting academics and in business practice. Many managers now use ABC to guide decisions and establish priorities. The ABC model is composed of both the cost assignment view and the process view with activities as the intersection of these two views [7]. The cost assignment view provides information about resources, activities, and cost objects. The process view provides financial and non-financial information about cost drivers and performance measures for each activity or process. Relying on ABC's analysis of how products consume resources, it models the causal relationship between products and resources used in their production. This enables ABC to provide an understanding of how costs are driven by the demands for activities within a process, and more accurate product cost information for evaluating the profitability of the firm's product lines [8].

The purpose of this paper is to develop an ABC joint products decision model incorporating capacity expansions and outsourcing features by using a mathematical programming approach, in order to lead to an optimal joint products further processing, capacity expansions or outsourcing decision. The remainder of this paper is organized into five sections. Sections 2 and 3 will detail the literature about the concepts of ABC and outsourcing. We develop our ABC model for joint products decision in Section 4. A numerical example is used to demonstrate how to apply the model in Section 5. Finally we present our conclusions in Section 6.

## 2. Concept of ABC

ABC techniques developed in practice and reported by Cooper and Kaplan [6] are seen as accurately assigning overhead costs to products. The detailed cost assignment view of ABC is shown in Fig. 1 [7,9]. ABC assumes that cost objects (e.g., products, product lines, processes, customers, channels, markets, etc.) create the need for activities, and activities create the need for resources. Accordingly, ABC uses a two-stage procedure to assign resource costs to cost objects. In the first stage, resource costs are assigned to various activities by resource drivers. Resource drivers are the factors chosen to approximate the consumption of resources by the activities. Each type of resource traced to

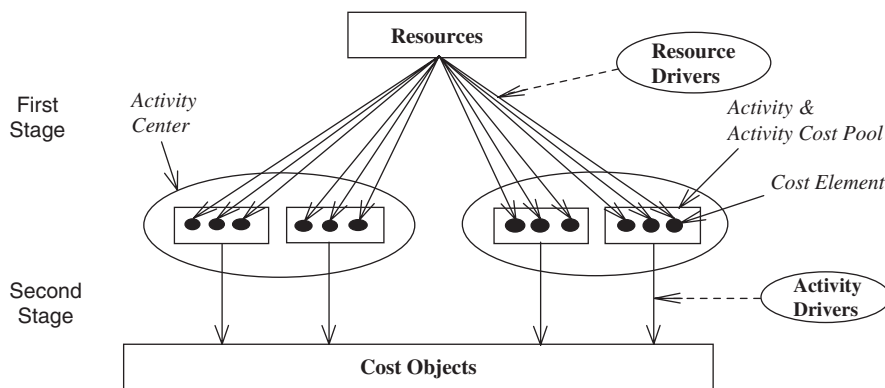


Fig. 1. The detailed cost assignment view of ABC. Source: Adapted from Turney [7, p. 97].

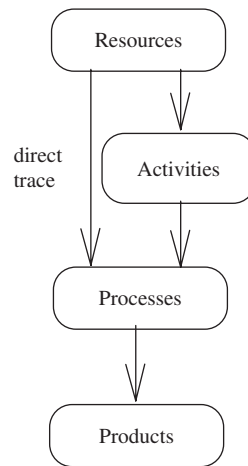


Fig. 2. Revised ABC model for joint products. Source: Tsai [19].

an activity becomes a cost element of an activity cost pool. Thus, an activity cost pool is the total cost associated with an activity. An activity center is composed of related activities, usually clustered by function or process. In the second stage, costs in each activity cost pool are distributed to cost objects by an activity driver which is used to measure the consumption of activities by the cost objects. Based on the characteristic of joint products we regard processes as the cost objects in this paper. The total cost of a specific process can be calculated by adding the costs of various activities assigned to that process.

Since 1988 ABC has evolved from the concept stage and has been widely used. Applications have been extended from manufacturing industries to service industries [10–12], non-profit organizations [13], and government bodies [14]. The information achieved through ABC cost assignment can be used for decisions concerning pricing, quoting, product mix [15], quality improvement [16], make versus buy, sourcing, product design, profitability analysis, and so on [7]. ABC frequently is applied together with other manufacturing management techniques, such as just-in-time and total quality management [10].

ABC also has been applied to various manufacturing systems [17,18], including the manufacturing system for joint products [19]. The product cost determination of joint products is based on processes. Thus, for joint product's costing, resource (production) costs should be traced eventually to processes, then to products. Some resource costs can be directly traced to processes, and some should be traced to processes by using activities as the intermediaries of cost assignment. Accordingly, Tsai [19] proposed the ABC model for joint products as shown in Fig. 2. In this revised ABC model, the joint product costs can be achieved by the following steps:

*Step 1:* Tracing direct resource costs to processes.

*Step 2:* Tracing indirect resource costs to activities.

*Step 3:* Tracing activity costs to processes.

*Step 4:* Tracing process costs to final products.

In this paper, we focus on steps 1–3 because the relevant information of joint products related decisions is process costs rather than joint products costs.

The resources used in manufacturing companies may include “people,” “machines,” “facilities,” and “utilities,” while the corresponding resource costs could be assigned to activities in the first stage of cost assignment view by using the resource drivers: “time,” “machine hours,” “square footage,” and “kilowatt hours,” respectively. The following are the categories for manufacturing activities: (1) unit-level activities (performed one time for one unit of product, e.g., machining, finishing), (2) batch-level activities (performed one time for a batch of products, e.g., setup, scheduling), (3) product-level activities (performed to benefit all units of a particular product, e.g., product design), and (4) facility-level activities (performed to sustain the manufacturing facility, e.g., plant guard and management) [20]. The costs of different levels of activities can be traced to products by using the different kinds of activity drivers in the second stage of cost assignment view. For example, “number of machine hours” is used for the activity “machining,” “setup hours” for “machine setup,” and “number of drawings” for “product design.” Usually, the costs of facility-level

activities cannot be traced to products with definite causal relationships and should be allocated to products with the appropriate allocation bases [21]. For the purpose of joint products decision, we divided facility-level activities into process-level activities (performed to benefit all products of a particular process) and facilities-level activities. Process-level activity costs of a specific process can be traced to that process. We assumed process-level activity costs of a specific process as the stepwise fixed cost and the costs of facility-level activities as the common fixed cost.

### 3. Outsourcing

Outsourcing is used to describe all subcontracting relationships between firms, and the hiring of workers in non-traditional jobs [22]. In practice, outsourcing is not only a “pure” make-or-buy decision, but also involves a switch from internal production to external procurement. For existing firms, outsourcing is always a de-integration decision, in which prior commitments to internal production should be ignored [3]. The strategic objective of outsourcing decision-makers should be to minimize the total costs of receiving any given quantity and quality of outsourced goods or service. The total costs consist of expenditures for the goods themselves and the costs associated with governing the outsourcing transaction.

Optimal choice between continued internal production and outsourcing requires more than mere consideration of production cost differences. According to transaction cost economics, the degree of asset specificity is an important consideration in the outsourcing decision. Outsourcing is only desirable when expected governance and coordination costs resulting from asset specific investments in the relationship with the future supplier are lower than the production cost saving advantage that the supplier may bring [2]. Specific assets are specialized to the exchange between buyer and seller rather than being usable for other purposes without losing value. For example, if outsourcing part of a production process requires the outsourcing firm to invest in dedicated transportation equipment, this investment is asset specific if it cannot be used for other purposes. Investments that can be put to other use without costs are not asset specific. Good outsourcing decisions require that decision-makers are appropriately sensitive to asset specific investments [3].

Perry [23] emphasizes that a firm could obtain the competitive advantages of reliability, quality and cost from contracting out the production of goods and services. Sharpe [24] argues that outsourcing arose to reduce the adjustment costs of responding to economic changes. Adjustments were a response to technological innovation, changing customer preference and other shifts in supply or demand. Glass and Saggi [25] find that outsourcing lowers the marginal cost of production, increases profit and creates greater incentives for innovation.

Thus, compared to capacity expansion, outsourcing provides a possible way to satisfy customer’s orders at the lower marginal costs also keeping the flexibility of operation in changing environments.

### 4. ABC joint products decision model with capacity expansions or outsourcing

Jaedicke [26] applied a linear programming (LP) technique to a cost-volume-profit (CVP) model, called “product mix” model in many management accounting or LP texts, which could aid management in determining the optimal product mix, maximizing total profit under some limits (constraints) to production or sales in the case of multi-product companies. In recent years, some authors have utilized various mathematical programming approaches to conduct the product-mix decision analysis under ABC [27–31] or under ABC with the capacity expansion features [15]. In this paper, we will extend their research to incorporating capacity expansions and outsourcing features into the joint products decision model under ABC.

#### 4.1. Assumptions

To develop a joint products decision model without loss of generality, we discuss a typical joint products production process as following. One unit of common input material processed in common process 0 will yield  $e_i$  units of joint products  $P_i$ . Joint products  $P_i$  can either be sold at the split-off point (we call it product  $P_{i0}$ ) or after processing further

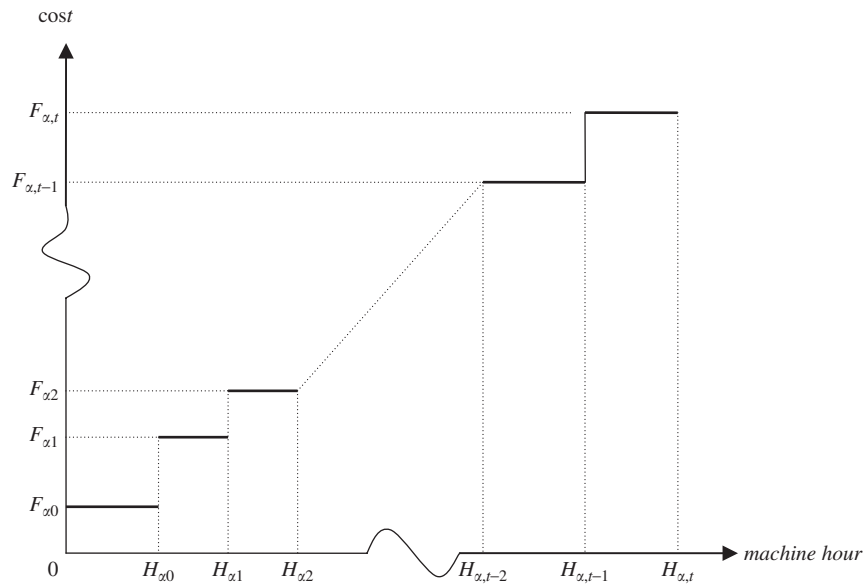


Fig. 3. Stepwise process-level activity cost of process  $\alpha$ .

in a separate process  $i$ , (we call it product  $P_{i1}$ ). One unit of joint products  $P_i$  processed in separate process  $i$  will yield  $f_i$  units of products  $P_{i1}$ .

In addition to the assumption about the production process, the ABC joint products decision model presented in this paper also has the following assumptions. First, the activities in a joint products company have been classified as unit-level, batch-level, product-level, process-level and facility-level activities, and the related resource drivers and activity drivers have been chosen by the company's ABC team through an ABC study. Second, the data on actual running activity cost per activity driver for each activity [32] have been collected and used in the model. Third, the unit selling prices and the unit direct material costs are constant within the relevant range. Fourth, the process-level activity cost of a specific process is regarded as the stepwise fixed cost that varies with machine hours. Fifth, renting or purchasing additional machines can expand machine hour resources. Sixth, using overtime work or additional night shifts with higher wage rates can expand direct labor resources. Seventh, the facility-level activity cost is regarded as the common fixed cost. Eighth, the firm is able to purchase products  $P_i$  and products  $P_{i1}$  from outside suppliers at a particular price. Ninth, the cost of asset specific from outsourcing is regarded as the stepwise fixed cost that varies with total outsourcing quantity.

#### 4.2. Capacity expansions and outsourcing features

According to the assumptions described above there are two capacity expansion features and one outsourcing feature in the ABC joint products decision model.

##### 4.2.1. Stepwise process-level activity cost

As shown in Fig. 3, the cost function of process-level activity cost of a specific process is a stepwise function that varies with machine hours, observed from a prior cost behavior analysis. The process-level activity cost of a specific process is  $F_{\alpha 0}$  under the current capacity  $H_{\alpha 0}$  machine hours. If the capacity is successively expanded to,  $H_{\alpha 1}$ ,  $H_{\alpha 2}$ ,  $\dots$ ,  $H_{\alpha t}$  machine hours, the process-level activity cost of a specific process increases to  $F_{\alpha 1}$ ,  $F_{\alpha 2}$ ,  $\dots$ ,  $F_{\alpha t}$ , respectively ( $\alpha = 0$  denotes common process 0;  $\alpha = 1, 2, \dots, n$  denotes separate process  $i$ ). Let  $Q_0$ ,  $q_{i1}/f_i$  be the quantity input in common process 0 and separate process  $i$ , respectively. Let  $\lambda_{0j}$  and  $\lambda_{ij}$  be the requirement of machine hours in the common process 0 and separate process  $i$  for processing one unit of input, respectively. As a result, the process-level activity cost of a specific process and the associated machine hour constraints are

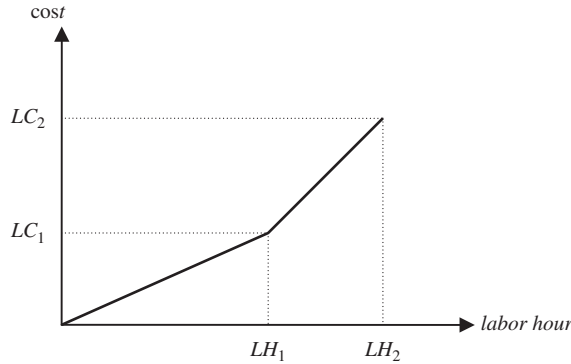


Fig. 4. Piecewise direct labor cost.

The process-level activity cost of specific process

$$= \sum_{k=0}^t F_{0k} \theta_{0k} + \sum_{i=1}^n \sum_{k=0}^t F_{ik} \theta_{ik}. \tag{1}$$

Constraints:

$$\lambda_{0j} Q_0 - \sum_{k=0}^t H_{0k} \theta_{0k} \leq 0, \tag{2}$$

$$\sum_{k=0}^t \theta_{0k} = 1, \tag{3}$$

$$\lambda_{ij} q_{i1} / f_i - \sum_{k=0}^t H_{ik} \theta_{ik} \leq 0, \quad i = 1, 2, \dots, n, \tag{4}$$

$$\sum_{k=0}^t \theta_{ik} = 1, \quad i = 1, 2, \dots, n, \tag{5}$$

where  $\theta_{0k}$  and  $\theta_{ik}$  are 0–1 variables. When  $\theta_{\alpha q} = 1 (q \neq 0)$ , we know that the capacity needs to be expanded to the  $q$ th level, i.e.,  $H_{\alpha q}$  machine hours. The first and second terms of Eq. (1) are the process-level activity costs of common process 0 and separate process  $i$ , respectively. Eqs. (2) and (4) describe machine hour constraints to the common process 0 and the separate process  $i$ , respectively.

#### 4.2.2. Piecewise direct labor cost

In this paper, we assume that using overtime work or additional night shifts with higher wage rates can expand direct labor resources. Thus, the total direct labor cost function will be a piecewise linear function as shown in Fig. 4. The available normal direct labor hour is  $LH_1$  and the direct labor hour can be expanded to  $LH_2$ ; the total direct labor cost is  $LC_1$  and  $LC_2$  at  $LH_1$  and  $LH_2$ , respectively. As a result, the total direct labor cost and the associated constraints are [33]

Total direct labor cost

$$= LC_1 a_1 + LC_2 a_2. \tag{6}$$

Constraints:

$$TL = LH_1a_1 + LH_2a_2, \quad (7)$$

$$a_0 - h_1 \leq 0, \quad (8)$$

$$a_1 - h_1 - h_2 \leq 0, \quad (9)$$

$$a_2 - h_2 \leq 0, \quad (10)$$

$$a_0 + a_1 + a_2 = 1, \quad (11)$$

$$h_1 + h_2 = 1, \quad (12)$$

where  $h_1$  and  $h_2$  are 0–1 variables within which exactly one variable must be non-zero;  $(a_0, a_1, a_2)$  is a set of non-negative variables within which at most two adjacent variables, in the order given to the set, can be non-zero [34,35];  $TL$  is the total direct labor hour we need and its function depending on the case under study.

If  $h_1 = 1$ , then  $h_2 = 0$  [from Eq. (12)],  $a_2 = 0$  [from Eq. (10)],  $a_0, a_1 \leq 1$  [from Eqs. (8) and (9)], and  $a_0 + a_1 = 1$  [from Eq. (11)]. Thus, from Eqs. (6) and (7) we know that total direct labor cost and total labor hour needed are  $LC_1a_1$  and  $LH_1a_1$ , respectively; this means that we will not need the overtime work.

If  $h_2 = 1$ , then  $h_1 = 0$  [from Eq. (12)],  $a_0 = 0$  [from Eq. (8)],  $a_1, a_2 \leq 1$  [from Eqs. (9) and (10)], and  $a_1 + a_2 = 1$  [from Eq. (11)]. Thus, from Eqs. (6) and (7) we know that total direct labor cost and total labor hour needed are  $LC_1a_1 + LC_2a_2$  and  $LH_1a_1 + LH_2a_2$ , respectively; this means that we will need the overtime work.

#### 4.2.3. Stepwise cost of asset specific from outsourcing

For an outsourcing decision, asset specific investments may include physical asset specific (e.g., transportation equipment), human asset specific (e.g., expenditures for bargaining, negotiation or monitoring) and so on [36]. Such costs usually vary with outsourcing quantity. We assume the cost function of asset specific from outsourcing is a stepwise function which varies with total outsourcing quantity and similar to the process-level activity cost. Let  $oq_{i0}$  and  $oq_{i1}$  be the outsourcing quantity of product  $P_i$  and  $P_{i1}$ , respectively. The cost of asset specific from outsourcing and the associated outsourcing quantity constraints are

$$\text{The cost of asset specific from outsourcing} = \sum_{s=0}^r \gamma_s AC_s. \quad (13)$$

$$\text{Constraints: } \sum_{i=1}^n (oq_{i0} + oq_{i1}) - \sum_{s=0}^r \gamma_s OQ_s \leq 0, \quad (14)$$

$$\sum_{s=0}^r \gamma_s = 1. \quad (15)$$

The cost of asset specific from outsourcing is \$0 ( $AC_0$ ) with no outsourcing ( $OQ_0 = 0, \gamma_0 = 1$ ). If the total outsourcing quantity is successively expanded to 1st, 2nd, ...,  $r$ th level, the cost of asset specific from outsourcing increases to  $AC_1, AC_2, \dots, AC_r$ , respectively. The maximum quantity of 1st, 2nd, ...,  $r$ th level is  $OQ_1, OQ_2, \dots, OQ_r$  units, respectively, where  $\gamma_s$  is 0–1 variable. When  $\gamma_s = 1$  ( $s \neq 0$ ), we know the total outsourcing quantity ( $oq_{i0} + oq_{i1}$ ) needs to be expanded to the  $s$ th level, i.e., between  $OQ_{s-1}$  and  $OQ_s$  units.

### 4.3. Description of the model

The ABC model for joint products decision with capacity expansions or outsourcing is as follows:

Max  $\pi$  = total revenue—total direct material cost of common process and separate processes—total direct labor cost—total unit-, batch-, product-, process- and facility-level activity costs of common process and separate processes—total cost of outsourcing

$$\begin{aligned}
 &= \left( \sum_{i=1}^n p_{i0}(q_{i0} + oq_{i0}) + \sum_{i=1}^n p_{i1}(q_{i1} + oq_{i1}) \right) - \left( m_0 Q_0 + \sum_{i=1}^n m_i q_{i1}/f_i \right) \\
 &\quad - (LC_1 a_1 + LC_2 a_2) \\
 &\quad - \left( \sum_{j \in U} d_j \lambda_{0j} Q_0 + \sum_{i=1}^n \sum_{j \in U} d_j \lambda_{ij} q_{i1}/f_i \right) - \left( \sum_{j \in B} d_j \beta_{0j} B_{0j} + \sum_{i=1}^n \sum_{j \in B} d_j \beta_{ij} B_{ij} \right) \\
 &\quad - \left( \sum_{j \in P} d_j \rho_{0j} R_0 + \sum_{i=1}^n \sum_{j \in P} d_j \rho_{ij} R_i \right) - \left( \sum_{k=0}^t F_{0k} \theta_{0k} + \sum_{i=1}^n \sum_{k=0}^t F_{ik} \theta_{ik} \right) - FC \\
 &\quad - \left( \sum_{i=1}^n oq_{i0} op_{i0} + \sum_{i=1}^n oq_{i1} op_{i1} + \sum_{s=0}^r \gamma_s AC_s \right)
 \end{aligned} \tag{16}$$

s.t.

(quantity of input and output constraint):

$$Q_0 = (q_{i0} + q_{i1}/f_i)/e_i, \quad i = 1, 2, \dots, n, \tag{17}$$

(piecewise unit-level direct labor constraints):

$$TL = LH_1 a_1 + LH_2 a_2, \tag{18}$$

$$a_0 - h_1 \leq 0, \tag{19}$$

$$a_1 - h_1 - h_2 \leq 0, \tag{20}$$

$$a_2 - h_2 \leq 0, \tag{21}$$

$$a_0 + a_1 + a_2 = 1, \tag{22}$$

$$h_1 + h_2 = 1, \tag{23}$$

(unit-level activity constraints):

$$\sum_{i=1}^n \lambda_{ij} q_{i1}/f_i + \lambda_{0j} Q_0 \leq M_j, \quad i = 1, 2, \dots, n; \quad j \in U, \tag{24}$$

(batch-level activity constraints):

$$\sum_{i=1}^n \beta_{ij} B_{ij} + \beta_{0j} B_{0j} \leq T_j, \quad i = 1, 2, \dots, n; \quad j \in B, \tag{25}$$

$$q_{i1}/f_i \leq b_{ij} B_{ij}, \quad i = 1, 2, \dots, n; \quad j \in B, \tag{26}$$

$$Q_0 \leq b_{0j} B_{0j}, \quad j \in B, \tag{27}$$



(product-level activity constraints):

$$\sum_{i=1}^n \rho_{ij} R_i + \rho_{0j} R_0 \leq V_j, \quad i = 1, 2, \dots, n; \quad j \in P, \quad (28)$$

$$q_{i1} \leq D_{i1} R_i, \quad i = 1, 2, \dots, n, \quad (29)$$

$$q_{i0} \leq D_{i0} R_0, \quad i = 1, 2, \dots, n, \quad (30)$$

$$q_{i0} + oq_{i0} \leq D_{i0}, \quad i = 1, 2, \dots, n, \quad (31)$$

$$q_{i1} + oq_{i1} \leq D_{i1}, \quad i = 1, 2, \dots, n, \quad (32)$$

$$R_0 - R_i \geq 0, \quad i = 1, 2, \dots, n, \quad (33)$$

(stepwise process-level machine hour constraints):

$$\lambda_{0j} Q_0 - \sum_{k=0}^t H_{0k} \theta_{0k} \leq 0, \quad (34)$$

$$\sum_{k=0}^t \theta_{0k} = 1, \quad (35)$$

$$\lambda_{ij} q_{i1} / f_i - \sum_{k=0}^t H_{ik} \theta_{ik} \leq 0, \quad i = 1, 2, \dots, n, \quad (36)$$

$$\sum_{k=0}^t \theta_{ik} = 1, \quad i = 1, 2, \dots, n, \quad (37)$$

(the cost of asset specific from outsourcing constraints):

$$\sum_{i=1}^n (oq_{i0} + oq_{i1}) - \sum_{s=0}^r \gamma_s O Q_s \leq 0, \quad (38)$$

$$\sum_{s=0}^r \gamma_s = 1, \quad (39)$$

$$\gamma_s: 0-1 \text{ variable}, \quad (40)$$

$$(a_0, a_1, a_2): \text{non-negative variables}, \quad (41)$$

$$(h_1, h_2): 0-1 \text{ variables}, \quad (42)$$

$$\theta_{0k}, \theta_{ik}: 0-1 \text{ variables}, \quad (43)$$

$$R_i: 0-1 \text{ variables}, \quad i = 1, 2, \dots, n, \quad (44)$$

$$B_{ij}: \text{non-negative integer variables}, \quad i = 1, 2, \dots, n, \quad j \in B, \quad (45)$$

where

$p_{i0}$  unit price of joint products  $P_i$  sold at the split-off point (the selling price of  $P_{i0}$ ),

$p_{i1}$  unit price of joint products  $P_i$  processed further in the separate process  $i$  after the split-off point (the selling price of  $P_{i1}$ ),

$Q_0$  quantity of direct material input in common process,

$q_i$  quantity of joint products  $P_i$  produced at the split-off point,

$q_{i0}$  quantity of joint products  $P_i$  sold at the split-off point (quantity sold of  $P_{i0}$ ),

$q_{i1}$	quantity of joint products $P_i$ sold after further processing in the separate processes $i$ (quantity sold of $P_{i1}$ ),
$op_{i0}$	outsourcing unit price of joint products $P_i$ ,
$op_{i1}$	outsourcing unit price of products $P_{i1}$ ,
$oq_{i0}$	outsourcing quantity of joint products $P_i$ for sale to customer,
$oq_{i1}$	outsourcing quantity of products $P_{i1}$ for sale to customer,
$e_i$	production coefficient of $Q_0$ and quantity produced of joint products $P_i$ (i.e., $q_i$ ),
$f_i$	production coefficient of joint products $P_i$ before and after further processing,
$m_0$	unit cost of direct material in the common process,
$m_i$	unit cost of direct material in the separate processes $i$ ,
$d_j$	the actual running activity cost per activity driver for activity $j$ ,
$\lambda_{0j}$	quantity of the activity driver of unit-level activity $j$ ( $j \in U$ ) consumed in the common process for processing one unit of input,
$\lambda_{ij}$	quantity of the activity driver of unit-level activity $j$ ( $j \in U$ ) consumed in the separate process $i$ for processing one unit of input,
$\beta_{0j}$	the requirement of the activity driver of batch-level activity $j$ ( $j \in B$ ) per batch for the common process,
$\beta_{ij}$	the requirement of the activity driver of batch-level activity $j$ ( $j \in B$ ) per batch for the separate process $i$ ,
$B_{0j}$	the number of batches of batch-level activity $j$ ( $j \in B$ ) for the common process,
$B_{ij}$	the number of batches of batch-level activity $j$ ( $j \in B$ ) for the separate process $i$ ,
$b_{0j}$	the number of units per batch of batch-level activity $j$ ( $j \in B$ ) for the common process,
$b_{ij}$	the number of units per batch of batch-level activity $j$ ( $j \in B$ ) for the separate process $i$ ,
$\rho_{0j}$	the requirement of the activity driver of product-level activity $j$ ( $j \in P$ ) for the common process,
$\rho_{ij}$	the requirement of the activity driver of product-level activity for the separate process $i$ ,
$R_0$	the indicator for producing joint products $P_i$ ( $R_0 = 1$ ) or not producing joint products $P_i$ ( $R_0 = 0$ ) in the common process,
$R_i$	the indicator for producing products $P_{i1}$ ( $R_i = 1$ ) or not producing products $P_{i1}$ ( $R_i = 0$ ) in the separate process $i$ ,
$M_j$	the capacity limit of the activity driver of unit-level activity $j$ ( $j \in U$ ),
$T_j$	the capacity limit of the activity driver of batch-level activity $j$ ( $j \in B$ ),
$V_j$	the capacity limit of the activity driver of product-level activity $j$ ( $j \in P$ ),
$D_{i0}$	the maximum demand for product $P_{i0}$ ,
$D_{i1}$	the maximum demand for product $P_{i1}$ ,
$FC$	the costs of facility-level activity.

Other variables and parameters are as mentioned before.

Eq. (16) represents the total profit function  $\pi$ . The first term of Eq. (16) denotes total revenues, the second term is total direct material cost of the common process and separate processes, the third is total labor cost, the fourth to the seventh represent the unit-level, batch-level, product-level and process-level activity costs of common process and separate processes, respectively, the eighth represents facility-level activity cost. The last term is the total cost of outsourcing. This ABC joint products decision model emphasizes the activity costs of processes. There is no need to allocate process costs to joint products.

Eq. (17) describes the relationship between quantity input and output. The input quantity of the common process is  $Q_0$  and the output quantities of joint products at the split-off point are  $q_i$  ( $i = 1, 2, 3, \dots, n$ ). The relation between  $Q_0$  and  $q_i$  can be written as follows:

$$q_i = Q_0 \times e_i, \tag{46}$$

where  $e_i$  denotes the production coefficient of  $Q_0$  and  $q_i$ . Joint products  $P_i$  can be sold at the split-off point (i.e., product  $P_{i0}$ ) or processed further in the separate process  $i$  (i.e., product  $P_{i1}$ ). The relation between  $q_{i1}$  and  $q_i - q_{i0}$  can be written as follows:

$$q_{i1} = (q_i - q_{i0}) \times f_i \quad \text{or} \quad q_{i1}/f_i = (q_i - q_{i0}), \tag{47}$$

where  $f_i$  is the production coefficient of  $q_{i1}$  and  $q_i - q_{i0}$  (i.e., production coefficient of joint products  $P_i$  before and after further processing).

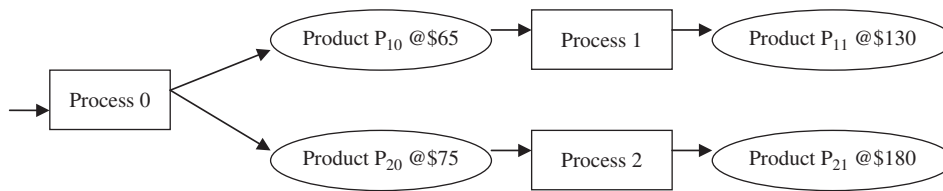


Fig. 5. The production process of company X.

Eq. (24) is the constraint associated with unit-level activities. The direct labor constraints described in Section 4.2.2 and the machine hour constraints described in Section 4.2.1. Eqs. (25)–(27) are the constraints associated with batch-level activities, where Eq. (25) is the capacity constraint for batch-level activity  $j$  ( $j \in B$ ). For example, we use “setup hours” as the activity driver of the batch-level activity “setup” because each process needs different setup hours. In this case,  $T_j$  is the available setup hours,  $\beta_{0j}$  and  $\beta_{ij}$  are the needed setup hours per batch for common process 0 and separate process  $i$ , respectively,  $B_{0j}$  and  $B_{ij}$  are the number of setup needed for common process 0 and separate process  $i$ , respectively,  $b_{0j}$  and  $b_{ij}$  are the average number of units in each setup batch for common process 0 and separate process  $i$ , respectively. In fact, there may be a different number of units in each setup batch for a specific process. However, we can use the average number of units for the purpose of planning.

Eqs. (28)–(33) are the constraints associated with product-level activities. Eq. (28) is the capacity constraint for product-level activity  $j$  ( $j \in P$ ). For example, we may use “number of drawings” as the activity driver of the product-level activity “product design”. In this case,  $V_j$  is the available number of drawings for the firm’s capacity, and  $\rho_{0j}$  and  $\rho_{ij}$  are the number of drawings needed for common process 0 and separate process  $i$ , respectively. Eqs. (29)–(32) are the market demand constraints. Eq. (33) represents the production order of common process 0 and separate process  $i$ . The companies have to produce joint products first, and then assess the desirability of further processing joint products beyond the split-off point. Thus, it is an impossible situation that  $R_0 = 0$  and  $R_i = 1$ .

## 5. Numerical example

In this section, we present a numerical example to illustrate the concepts described in the previous section. First, we obtain the optimal joint products decision under current capacity. Then, we consider the capacity expansions. Finally, we analyze the optimal joint products decision with capacity expansions or outsourcing.

Assume that the production function of Company X is such that one unit of material input processed in process 0 will yield one unit of product  $P_1$  and one unit of product  $P_2$ . Product  $P_1$  can be sold at the split-off point for \$65 per unit (i.e., product  $P_{10}$ ) or processed further in process 1 then sold at a price of \$130 (i.e., product  $P_{11}$ ). One unit of product  $P_1$  process further in process 1 will yield one unit of product  $P_{11}$ . Product  $P_2$  can be sold at the split-off point for \$75 per unit (i.e., product  $P_{20}$ ), or processed further in process 2 then sold at a price of \$180 (i.e., product  $P_{21}$ ). One unit of product  $P_2$  processed further in process 2 will yield one unit of product  $P_{21}$ . Fig. 5 shows the production process of Company X.

Company X needs the following main activities in producing these joint products: two unit-level activities, machining and finishing ( $U = \{1, 2\}$ ), two batch-level activities, scheduling and setup ( $B = \{3, 4\}$ ), one product-level activity, product design ( $P = \{5\}$ ), three process-level activities, machine 0, machine 1 and machine 2 for process 0, process 1 and process 2, respectively. The data of activity cost of each process and available capacity are presented in Table 1.

From Table 1, we know that the process-level activity costs of process 0, process 1, process 2 are  $F_{00} = \$2,000,000$ ,  $F_{10} = \$100,000$ ,  $F_{20} = \$720,000$  under the current capacity  $H_{00} = 200,000$ ,  $H_{10} = 20,000$ ,  $H_{20} = 60,000$  machine hours, respectively, and that the capacity can be expanded to  $H_{01} = 250,000$ ,  $H_{11} = 40,000$ ,  $H_{21} = 75,000$ , or  $H_{02} = 300,000$ ,  $H_{12} = 60,000$ ,  $H_{22} = 90,000$  machine hours by renting or purchasing additional machines with the process-level activity costs increasing to  $F_{01} = \$2,500,000$ ,  $F_{11} = \$220,000$ ,  $F_{21} = \$900,000$  or  $F_{02} = \$3,200,000$ ,  $F_{12} = \$350,000$ ,  $F_{22} = \$1,100,000$ , respectively. The available normal direct labor hour is  $LH_1 = 300,000$  h with the normal wage rate of \$2/h and the direct labor hour can be expanded to  $LH_2 = 600,000$  h with the overtime wage rate of \$3/h. Further, assume that two unit-level activities, machining and finishing, need direct labor. Machining activity needs  $1/2\lambda_{01}$ ,  $1/2\lambda_{11}$  and  $1/2\lambda_{21}$  labor hour for process 0, process 1 and process 2, respectively, to process one unit of

Table 1  
Example data

			Process 0		Process 1	Process 2	Available capacity
			Product $P_{10}$	Product $P_{20}$	Product $P_{11}$	Product $P_{21}$	
Maximum demand	$D_{i0}, D_{i1}$		30,000	30,000	30,000	30,000	
Selling price	$P_{i0}, P_{i1}$		\$65	\$75	\$130	\$180	
Production coefficient	$e_i, f_i$		1	1	1	1	
Direct material	$m_0, m_i$			\$30	\$20	\$10	
	Activity	cost/driver					
<i>Unit-level activity</i>	$j$ Driver	$d_j$					
Machining	1 Machine hours	$\lambda_{01}, \lambda_{i1}$	5		2	3	
Finishing	2 Labor hours	$\lambda_{02}, \lambda_{i2}$	4		1	2	
<i>Batch-level activity</i>							
Scheduling	3 Production Orders	\$5,000 $\beta_{03}, \beta_{i3}$	1		1	1	$T_3 = 70$
		$b_{03}, b_{i3}$	3500		2500	1000	
Setup	4 Setup Hours	\$20,000 $\beta_{04}, \beta_{i4}$	4		1	2	$T_4 = 90$
		$b_{04}, b_{i4}$	5000		2000	2500	
<i>Product-level activity</i>							
Design	5 Drawings	\$10,000 $\rho_{05}, \rho_{i5}$	30		10	20	$V_5 = 75$
<i>Process-level activity</i>			Machine 0		Machine 1	Machine 2	
<i>Current capacity</i>							
Cost			$F_{00} = \$2,000,000$		$F_{10} = \$100,000$	$F_{20} = \$720,000$	
Machine hours			$H_{00} = 200,000$		$H_{10} = 20,000$	$H_{20} = 60,000$	
<i>Capacity expansion-1</i>							
Cost			$F_{01} = \$2,500,000$		$F_{11} = \$220,000$	$F_{21} = \$900,000$	
Machine hours			$H_{01} = 250,000$		$H_{11} = 40,000$	$H_{21} = 75,000$	
<i>Capacity expansion-2</i>							
Cost			$F_{02} = \$3,200,000$		$F_{12} = \$350,000$	$F_{22} = \$1,100,000$	
Machine hours			$H_{02} = 300,000$		$H_{12} = 60,000$	$H_{22} = 90,000$	
Facility-level cost	Plant guard and management				\$200,000		
<i>Direct labor constraint</i>							
Cost	$LC_1 = \$600,000$	$LC_2 = \$1,500,000$					
Labor hours	$LH_1 = 300,000$	$LH_2 = 600,000$					
Wage rate	$r_1 = \$2/h$	$r_2 = \$3/h$					
Outsourcing data:	Outsourcing price		$op_{10} = \$60$		$op_{20} = \$70$	$op_{11} = \$100$	$op_{21} = \$150$
Cost of asset specific from outsourcing			Total outsourcing quantity ( $oq_{10} + oq_{20} + oq_{11} + oq_{21}$ )			$OQ_s$	
$AC_0 = \$0$			$oq_{10} + oq_{20} + oq_{11} + oq_{21} = 0$			$OQ_0 = 0$	
$AC_1 = \$70,000$			$1 \leq oq_{10} + oq_{20} + oq_{11} + oq_{21} \leq 20,000$			$OQ_1 = 20,000$	
$AC_2 = \$100,000$			$20,001 \leq oq_{10} + oq_{20} + oq_{11} + oq_{21} \leq 50,000$			$OQ_2 = 50,000$	

input. It means that the two activities utilize the same group of multi-function workers. Thus, Eq. (18) will be

$$(1/2\lambda_{01} + \lambda_{02})Q_0 + \sum_{i=1}^n (1/2\lambda_{i1} + \lambda_{i2})q_{i1} = LH_1a_1 + LH_2a_2. \tag{48}$$

5.1. The optimal joint products decision under current capacity

Assume that Company X has to decide the optimal quantity produced of joint products with current capacity. By using Eqs. (16)–(48), let  $\theta_{00} = 1, \theta_{10} = 1, \theta_{20} = 1$  and ignoring the variables and parameters about outsourcing, the example is formulated as follows:

$$\begin{aligned} \text{Max } \pi = & (65q_{10} + 75q_{20} + 130q_{11} + 180q_{21}) - (30Q_0 + 20q_{11} + 10q_{21}) - (600,000a_1 \\ & + 1,500,000a_2) - (5000B_{03} + 5000B_{13} + 5000B_{23}) \\ & - (80,000B_{04} + 20,000B_{14} + 40,000B_{24}) - (300,000R_0 + 100,000R_1 + 200,000R_2) \\ & - (2,000,000 + 100,000 + 720,000) - 200,000 \end{aligned}$$

s.t.

(quantity of input and output constraint):

$$Q_0 = q_{10} + q_{11} = q_{20} + q_{21},$$

(piecewise unit-level direct labor constraints):

$$6.5Q_0 + 2q_{11} + 3.5q_{21} = 300,000a_1 + 600,000a_2,$$

$$a_0 - h_1 \leq 0, \quad a_1 - h_1 - h_2 \leq 0, \quad a_2 - h_2 \leq 0, \quad a_0 + a_1 + a_2 = 1, \quad h_1 + h_2 = 1,$$

(batch-level activity constraints):

$$\text{Scheduling: } B_{03} + B_{13} + B_{23} \leq 70,$$

$$Q_0 - 3000B_{03} \leq 0, \quad q_{11} - 2000B_{13} \leq 0, \quad q_{21} - 1000B_{23} \leq 0,$$

$$\text{Setup: } 4B_{04} + B_{14} + 2B_{24} \leq 90,$$

$$Q_0 - 5000B_{04} \leq 0, \quad q_{11} - 2000B_{14} \leq 0, \quad q_{21} - 2500B_{24} \leq 0,$$

(product-level activity constraints):

$$30R_0 + 10R_1 + 20R_2 \leq 75,$$

$$q_{10} - 30,000R_0 \leq 0, \quad q_{20} - 30,000R_0 \leq 0, \quad q_{11} - 30,000R_1 \leq 0, \quad q_{21} - 30,000R_2 \leq 0,$$

$$R_0 - R_1 \geq 0, \quad R_0 - R_2 \geq 0,$$

(process-level machine hour constraints):

$$5Q_0 - 200,000 \leq 0, \quad 2q_{11} - 20,000 \leq 0, \quad 3q_{21} - 60,000 \leq 0,$$

where  $Q_0, q_{10}, q_{11}, q_{20}, q_{21}, B_{03}, B_{13}, B_{23}, B_{04}, B_{14}, B_{24} \geq 0$  and will be an integer.  $R_0, R_1, R_2, h_1, h_2$  are 0–1 variables, and  $a_0, a_1, a_2 \geq 0$ . This is a mixed-integer programming (MIP) model. We solve this problem by utilizing the software, LINGO 8.0, and obtain the following optimal solution:

$$Q_0 = 40,000, \quad q_{10} = 30,000, \quad q_{20} = 20,000, \quad q_{11} = 10,000, \quad q_{21} = 20,000,$$

$$a_0 = 0, \quad a_1 = 0.83, \quad a_2 = 0.17,$$

$$h_1 = 0, \quad h_2 = 1, \quad R_0 = 1, \quad R_1 = 1, \quad R_2 = 1,$$

$$B_{03} = 14, \quad B_{13} = 5, \quad B_{23} = 20,$$

$$B_{04} = 8, \quad B_{14} = 5, \quad B_{24} = 8.$$

Accordingly, the optimal quantity of joint product produced is  $(q_{10}, q_{20}, q_{11}, q_{21}) = (30,000, 20,000, 10,000, 20,000)$ , which requires 350,000 ( $=6.5 \times 40,000 + 2 \times 10,000 + 3.5 \times 20,000$ ) direct labor hours. The total profit  $\pi$  is \$1,125,000. Because the customer's demand for  $P_{20}, P_{11}$ , and  $P_{21}$  are not fully satisfied, the Company X can consider either expanding capacity or outsourcing to satisfy the customer's order and maximize total profit.

## 5.2. The optimal joint products decision with capacity expansions

Assume that Company X has to decide the optimal quantity produced of joint products with capacity expansions. By using Eqs. (16)–(48) and ignoring the variables and parameters about outsourcing, the example is formulated as follows:

$$\begin{aligned} \text{Max } \pi = & (65q_{10} + 75q_{20} + 130q_{11} + 180q_{21}) - (30Q_0 + 20q_{11} + 10q_{21}) - (600,000a_1 + 1,500,000a_2) \\ & - (5000B_{03} + 5000B_{13} + 5000B_{23}) - (80,000B_{04} + 20,000B_{14} + 40,000B_{24}) \\ & - (300,000R_0 + 100,000R_1 + 200,000R_2) - (2,000,000\theta_{00} \\ & + 2,500,000\theta_{01} + 3,200,000\theta_{02} + 100,000\theta_{10} + 220,000\theta_{11} \\ & + 350,000\theta_{12} + 720,000\theta_{20} + 900,000\theta_{21} + 1,100,000\theta_{22}) \\ & - 200,000 \end{aligned}$$

s.t.

quantity of input and output constraint, piecewise unit-level direct labor constraints, batch-level activity constraints and product-level activity constraints which were described in Section 5.1, there are additional constraints as follow:

(process-level machine hour constraints)

$$\begin{aligned} 5Q_0 - 200,000\theta_{00} - 250,000\theta_{01} - 300,000\theta_{02} &\leq 0, & \theta_{00} + \theta_{01} + \theta_{02} &= 1, \\ 2q_{11} - 20,000\theta_{10} - 40,000\theta_{11} - 60,000\theta_{12} &\leq 0, & \theta_{10} + \theta_{11} + \theta_{12} &= 1, \\ 3q_{21} - 60,000\theta_{20} - 75,000\theta_{21} - 90,000\theta_{22} &\leq 0, & \theta_{20} + \theta_{21} + \theta_{22} &= 1, \end{aligned}$$

where  $\theta_{00}, \theta_{01}, \theta_{02}, \theta_{10}, \theta_{11}, \theta_{12}, \theta_{20}, \theta_{21}$  and  $\theta_{22}$  are 0–1 variables, and other variables and parameters are as described in Section 5.1. The optimal solution is as follows:

$$\begin{aligned} Q_0 &= 60,000, & q_{10} &= 30,000, & q_{20} &= 30,000, & q_{11} &= 30,000, & q_{21} &= 30,000, \\ a_0 &= 0, & a_1 &= 0.15, & a_2 &= 0.85, & R_0 &= 1, & R_1 &= 1, & R_2 &= 1, \\ h_1 &= 0, & h_2 &= 1, & \theta_{00} &= 0, & \theta_{01} &= 0, & \theta_{02} &= 1, \\ B_{03} &= 20, & B_{13} &= 15, & B_{23} &= 30, & \theta_{10} &= 0, & \theta_{11} &= 0, & \theta_{12} &= 1, \\ B_{04} &= 12, & B_{14} &= 15, & B_{24} &= 12, & \theta_{20} &= 0, & \theta_{21} &= 0, & \theta_{22} &= 1. \end{aligned}$$

Accordingly, the optimal quantity of joint product produced is  $(q_{10}, q_{20}, q_{11}, q_{21}) = (30,000, 30,000, 30,000, 30,000)$ , which requires 555,000 ( $=6.5 \times 60,000 + 2 \times 30,000 + 3.5 \times 30,000$ ) direct labor hours and 300,000 machine hours, 60,000 machine hours and 90,000 machine hours of machine 0, machine 1 and machine 2, respectively ( $5 \times 60,000, 2 \times 30,000, 3 \times 30,000$ ). The total profit  $\pi$  is \$1,920,000. This means that the machine capacity of process 0, process 1 and process 2 is expanded to the second level by purchasing or renting additional machines. Company X can satisfy the customer’s order and increase total profit by capacity expansions. However, the operation risk will increase too. The other possible way to resolve this problem, with appropriate risk control, is outsourcing.

5.3. The optimal joint products decision with capacity expansions or outsourcing

Consider that Company X has to decide the optimal quantity produced of joint products with capacity expansions or outsourcing. By using Eqs. (16)–(48), the example is formulated as follows:

$$\begin{aligned} \text{Max}\pi &= [65(q_{10} + oq_{10}) + 75(q_{20} + oq_{20}) + 130(q_{11} + oq_{11}) + 180(q_{21} + oq_{21})] - (30Q_0 + 20q_{11} + 10q_{21}) \\ &\quad - (600,000a_1 + 1,500,000a_2) - (5000B_{03} + 5000B_{13} + 5000B_{23}) \\ &\quad - (80,000B_{04} + 20,000B_{14} + 40,000B_{24}) - (300,000R_0 + 100,000R_1 + 200,000R_2) \\ &\quad - (2,000,000\theta_{00} + 2,500,000\theta_{01} + 3,200,000\theta_{02} + 100,000\theta_{10} \\ &\quad + 220,000\theta_{11} + 350,000\theta_{12} + 720,000\theta_{20} + 900,000\theta_{21} + 1,100,000\theta_{22}) \\ &\quad - 200,000 \\ &\quad - (oq_{10} \times 60 + oq_{20} \times 70 + oq_{11} \times 100 + oq_{21} \times 150) - (70,000\gamma_1 + 100,000\gamma_2) \end{aligned}$$

s.t.

quantity of input and output constraint, piecewise unit-level direct labor constraints, batch-level activity constraints, product-level activity constraints and process-level machine hour constraints which were described in Section 5.2, there are additional constraints as follow:

(product-level activity constraints):

$$q_{10} + oq_{10} \leq 30,000, \quad q_{20} + oq_{20} \leq 30,000, \quad q_{11} + oq_{11} \leq 30,000, \quad q_{21} + oq_{21} \leq 30,000,$$

(the cost of asset specific from outsourcing constraint):

$$\begin{aligned} (oq_{10} + oq_{20} + oq_{11} + oq_{21}) - (20,000\gamma_1 + 50,000\gamma_2) &\leq 0, \\ \gamma_0 + \gamma_1 + \gamma_2 &= 1, \end{aligned}$$

where  $\gamma_0, \gamma_1, \gamma_2$  are 0–1 variables, and other variables and parameters are as described in Section 5.1 and 5.2. The optimal solution is as follows:

$$\begin{aligned} Q_0 &= 50,000, & q_{10} &= 30,000, & q_{20} &= 25,000, & q_{11} &= 20,000, & q_{21} &= 25,000, \\ oq_{10} &= 0, & oq_{20} &= 5000, & oq_{11} &= 10,000, & oq_{21} &= 5000, \\ a_0 &= 0, & a_1 &= 0.492, & a_2 &= 0.508, & R_0 &= 1, & R_1 &= 1, & R_2 &= 1, \\ h_1 &= 0, & h_2 &= 1, & \theta_{00} &= 0, & \theta_{01} &= 1, & \theta_{02} &= 0, \\ B_{03} &= 17, & B_{13} &= 10, & B_{23} &= 25, & \theta_{10} &= 0, & \theta_{11} &= 1, & \theta_{12} &= 0, \\ B_{04} &= 10, & B_{14} &= 10, & B_{24} &= 10, & \theta_{20} &= 0, & \theta_{21} &= 1, & \theta_{22} &= 0, \\ \gamma_0 &= 0, & \gamma_1 &= 1, & \gamma_2 &= 0. \end{aligned}$$

Accordingly, the optimal quantity of joint product produced is  $(q_{10}, q_{20}, q_{11}, q_{21}) = (30,000, 25,000, 20,000, 25,000)$ , which requires 452,500 ( $=6.5 \times 50,000 + 2 \times 20,000 + 3.5 \times 25,000$ ) direct labor hours and 250,000 machine hours, 40,000 machine hours and 75,000 machine of machine 0, machine 1 and machine 2, respectively ( $5 \times 50,000, 2 \times 20,000, 3 \times 25,000$ ). The total profit  $\pi$  is \$2,042,500. This means that the machine capacity of process 0, process 1 and process 2 is expanded to the first level by purchasing or renting additional machines. The outsourcing quantity is  $(oq_{10}, oq_{20}, oq_{11}, oq_{21}) = (0, 5000, 10,000, 5000)$ . Company X can satisfy the customer's order and increase total profit by capacity expansions and outsourcing part of products. The combination of capacity expansion and outsourcing leads to an optimal joint products decision with higher total profit and appropriate control of operational risk.

## 6. Conclusion

In recent years, outsourcing has become an important strategy for many business organizations. For a successful outsourcing decision, the advantage of cost savings is an important consideration. Thus, the decision about outsourcing requires an accurate analysis of relevant costs. Application of the ABC model to joint products to trace resource costs to processes can improve the accuracy of product (processes) cost data derived from the traditional volume-based or unit-based costing systems. One of the special features of ABC is that it uses both volume-based (i.e., unit-level) and non-volume-based (i.e., batch-level, product-level, or facility-level) drivers to assign activity costs to products according to the nature of the activities. Since the relevant information of joint products related decisions is process costs rather than joint products costs, the ABC joint products decision model that this paper proposes emphasizes the activity costs of processes. There is no need to allocate process costs to joint products. Furthermore, to extend the existing research literature, this paper incorporates capacity expansion and outsourcing features into the ABC joint products decision model by using a mathematical programming approach.

In order to maximize total profit and satisfy the customer's order under the situation in which there is lack of sufficient capacity to meet market demand, the firms who produce joint products have to make decisions about further processing, capacity expansions or outsourcing. An ABC model for joint products related decisions is presented in this paper and a numerical example is used to demonstrate how to apply the model under three different conditions. Through accurate analysis of relevant costs, the firms can maintain the equilibrium of internal production and outsourcing, and also obtain a competitive advantage from outsourcing by being able to allocate their own resources efficiently.

This paper contributes to the management sciences by developing a new mixed integer programming joint products decision model that maximizes a firm's profit with following major types of ABC constraints: (1) unit-level piecewise direct labor constraints, (2) batch-level activity constraints, (3) product-level activity constraints, and (4) stepwise process-level machine hour constraints. With the model presented in this paper, we could evaluate the comparative benefit of expanding the various kinds of capacity and outsourcing simultaneously. By applying this model, the companies who produce joint products will be able to make optimal decisions about further processing, capacity expansions or outsourcing.

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