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# Embedding management discretionary power into an ABC model for a joint products mix decision

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#### ABSTRACT

Activity-based costing (ABC) and Theory of Constraints (TOC) are popular managerial tools for evaluating product mix decisions. A challenging aspect of the product mix problem involves evaluating joint products. These products involve complex interactions among resources and products and sequential decisions concerning producing joint products and their further processing. ABC and TOC assume that managers either have complete control or have no control over labor and overhead resources. However, managers generally have varying degrees of control over these resources. The purpose of this paper is to develop a general ABC model that embeds management discretionary power over labor and overhead resources. The paper illustrates the general model leads to optimal joint product mix decisions when the ABC and the TOC may not.

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#### 1. Introduction

"Joint production" is the term used in economics to describe situations, where as a result of a single process, two or more products are made. In other words, when two or more products are jointly produced in a common manufacturing process, they are called joint products. This situation is distinguished from the more common multiple production, in which a number of different products are made by different processes in the same facilities. Many companies, such as petroleum refiners, lumber mills, meat packers or flour mills, produce a multitude of products in their manufacturing processes. Joint products are produced simultaneously by a joint process or series of processes. All costs incurred before the split-off point of joint products are referred to as joint costs, and costs incurred for further processing and disposal are referred

to as separable costs. Due to capacity constraints, when joint products can either be sold at the split-off point or after further separate processing, the companies who produce joint products have to decide which joint products should be sold at the split-off point and which should be sold after further processing. The decision can be viewed as the joint products mix.

One of the most important decisions made in production systems is determining the most profitable products. Activity-based costing (ABC) has been suggested by many articles to guide such decisions and to establish priorities (e.g. Tsai and Lin, 2004). ABC provides management with an understanding of how costs are driven by the demands for activities within a process. There has also been a lot of research into applying the Theory of Constraints (TOC) to optimize product mix decisions (e.g. Plenert, 1993). TOC-based decisions maximize throughput (or contribution margin) under process constraints (Jayson et al., 1987). Souren et al. (2005) analyzed the deficiencies of the TOC for product mix decisions. One of their conclusions is that the TOC-based approach may lead to a non-optimal

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solution if joint material costs are used in producing the firm's products. The use of joint material involves a production process where two or more products emerge from a production process whether the firm wants each of the products or not. The production of products from a joint production process is a special case of the more general product mix problem.

The literature, which has focused on joint products mix, is not extensive. Hartley (1971) presents several cases of linear programming (LP) formulations to perform comprehensive analysis of joint products decisions. In his examples, they are constrained by equipment capacity, market demands and limited material availability. As compared with product mix decisions, joint products mix has two special features. First, the joint products are jointly produced at the same time and during the same process. That is the quantities produced of each joint product cannot be decided individually. Second, each joint product and its further processing product must be produced in sequence. In other words, joint products have to be produced first and then the desirability of further processing has to be assessed.

In the light of this, this paper examines the determination of an optimal product mix from the production of joint products with TOC and ABC models under different economic conditions. The remainder of this paper is organized into four sections. Section 2 will detail the literature about the product mix models of TOC and ABC. We revise the TOC and ABC models and develop a general ABC model for joint products mix in Section 3. A numerical example is used to demonstrate how to solve these models with mix-integer programming (MIP) in Section 4. Finally, a conclusion is provided in Section 5.

## 2. Concepts of ABC and TOC

Traditional cost accounting, which mainly uses volume-related allocation bases (such as direct labor hours or direct labor costs) to allocate overhead costs, can systematically distort product costs in advanced manufacturing environments when overhead costs are a significant portion of overall costs. ABC techniques

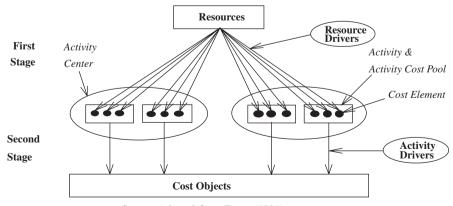
developed in practice and reported by Cooper and Kaplan (1988) are seen as accurately assigning overhead costs to products.

Fig. 1 shows the detailed cost assignment view of ABC. It assumes that cost objects (products, product lines, processes, customers, channels, markets, and so on) create the need for activities, and activities create the need for resources. Accordingly, the technique uses a two-stage procedure to assign resource costs to cost objects.

In the first stage, resource costs are assigned to various activities using resource drivers. Resource drivers are the factors selected to approximate resources consumed by various activities. Each type of resource traced to an activity becomes a cost element of an activity cost pool. An activity cost pool is thus the total cost associated with an activity. An activity center comprises related activities, usually clustered by function or process. In the second stage, each activity cost pool is distributed to cost objects using an adequate activity driver that measures activity consumption by cost objects (Turney, 1991).

The resources used in manufacturing companies may include "people," "machines," "facilities," and "utilities," while the corresponding resource costs could be assigned to activities in the first stage of the cost assignment view by using the resource drivers "time," "machine hours," "square footage," and "kilowatt hours," respectively. The following are the categories for manufacturing activities: (1) unit-level activities, (2) batch-level activities, (3) product-level activities, and (4) facility-level activities (Cooper, 1990). The costs of different levels of activities can be traced to products by using different kinds of activity drivers in the second stage of the cost assignment view. For example, "number of machine hours" is used for the activity "machining," "setup hours" for "machine setup," and "number of drawings" for "product design" (Tsai, 1996a). Usually, the costs of facility-level activities cannot be traced to products with definite causal relationships and should be allocated to products with the appropriate bases (Metzger, 1993).

Since 1988, ABC has evolved from the concept stage and has been widely used. Applications extend from manufacturing industries (Singer and Donoso, 2008) to service industries (Carlson and Young, 1993; Chan, 1993;



Source: Adapted from Turney(1991)

Fig. 1. Detailed cost assignment view of ABC.

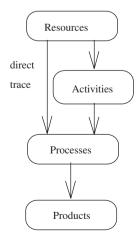
Tsai and Kuo, 2004; Tsai and Hsu, 2008), non-profit organizations (Antos, 1992), and government bodies (Harr, 1990). The information achieved through ABC cost assignment can be used for decisions concerning pricing, quoting, product mix (Tsai and Lin, 2004; Kee, 2008), quality improvement (Tsai, 1998), make versus buy, outsourcing (Tsai and Lai, 2007; Tsai et al., 2007a), product design and development (Qian and Ben-Arieh, 2008), financial and physical flows analysis (Comelli et al., 2008), and environmental management (Tsai and Hung, 2008a, b). ABC is frequently applied together with other management concepts or techniques, such as total quality management (Carlson and Young, 1993), game theory (Charles and Hansen, 2008) and inventory control systems (Berling, 2008).

ABC also has been applied to various manufacturing systems (Dhavale, 1993; Zhuang and Burns, 1992), including the manufacturing system for joint products (Tsai, 1996b). The product cost determinations of joint products is based on processes. Thus, for joint product's costing, resource (production) costs should be traced to processes, then to products. Some resource costs can be directly traced to processes, and some should be traced to processes by using activities as the intermediums of cost assignment. Accordingly, Tsai (1996b) proposed the ABC model for joint products as shown in Fig. 2. In this revised ABC model, the joint product costs can be achieved by the following steps:

- Step 1. Tracing direct resource costs to processes.
- Step 2. Tracing indirect resource costs to activities.
- Step 3. Tracing activity costs to processes.

*Step 4.* Tracing process costs (including direct and indirect resource costs) to final products.

The purpose of Tsai (1996b) is to calculate the costs of joint products for general costing. However, allocating joint costs to products for decision making is inappropriate and not necessary. Owing to the relevant information of joint products mix decisions is process costs rather than joint products costs, we will focus on steps 1–3 in this



Source:Adapted fromTsai (1996b)

Fig. 2. Revised ABC model for joint products.

paper. In other words, we don't allocate process costs to joint products for decision making.

In addition to ABC, the product mix problem also has been discussed in many TOC literatures. TOC is a management philosophy that bases priorities on the factors that constrain the process of providing goods and/or services to customers. It was developed by Goldratt (1990) as a process of continuous improvement. Goldratt indicates that many of the assumptions underlying traditional costbased accounting systems are no longer valid and that these systems are leading many companies to disaster. Consequently, he proposes using an alternative measurement system to evaluate the impact of production-related decisions. Under this system, direct material is treated as a variable cost. Conversely, labor and overhead are assumed to be resources the firm is committed to acquiring and is unable to influence. Therefore, the cost of labor and overhead supplied to production is treated as a period expense.

Researchers have examined linear programming (LP) and TOC for evaluating product mix decisions. Patterson (1992), focusing on the Throughput Accounting (TA) concept, linked product mix decisions directly to the TOC philosophy and used the ratio of contribution margin to the processing time on the bottleneck as the production priority. Plenert (1993) applied the integer LP approach to a TOC product mix example with multiple constrained resources. Tsai et al. (2007b) developed a TOC-based algorithm for obtaining the optimal joint products mix decision. In this paper, we also apply the TOC approach to fit the two special features, the jointly produced characteristic of the joint products and the sequentially produced characteristic between each joint product and its further processing product, to obtain an optimal joint products mix.

## 3. Models for joint products mix

In this section, we discuss three models for joint products mix. According to special features of the joint products mix, we classify the activity drivers in these three models as follows:

- (1) unit-level activities: performed one time for one unit of input in a joint process or in separate processes, e.g., material processing, finishing.
- (2) batch-level activities: performed one time for a batch input in a joint process or in separate processes, e.g., setup.
- (3) process-level activities: performed to benefit all units of a particular product in a joint process or in separate processes, e.g., process design.

We dropped facility-level activities because of they performed to sustain the manufacturing facility. Facility-level activities were not included in the model of a joint product mix problem since they relate to the firm as a whole and their cost cannot be traced directly to products.

#### 3.1. The TOC-based model

The model of assessment of the desirability of further processing joint products beyond the split-off point with the TOC may be expressed as

$$\max Z = \sum_{i=1}^{n} p_{i0} q_{i0} + \sum_{i=1}^{n} p_{i1} q_{i1}$$
$$- \sum_{i \in U,B,P} \sum_{k=1}^{m} C_{jk} V_{jk} - \sum_{i=1}^{n} m_i q_{i1} / a_i - m_0 Q_0$$
(1)

S.T. 
$$Q_0 = (q_{i0} + q_{i1}/a_i)/e_i$$
  $i = 1, 2, 3, ..., n$  (2)

$$\sum_{i=1}^{n} X_{ijk} q_{i1} / a_i + X_{0jk} Q_0 \leqslant V_{jk} \quad k = 1, 2, \dots, m$$

$$Q_0, q_{i0}, q_{i1} \geqslant 0$$
(3)

where Z is the value of object function (total profits),  $p_{i0}$ the unit price of joint products  $P_i$  sold at the split-off point (the selling price of  $P_{i0}$ ),  $p_{i1}$  the unit price of joint products  $P_i$  processed further in the separate process i after the split-off point (the selling price of  $P_{i1}$ ),  $Q_0$  the quantity of direct material input in common process,  $q_i$  the quantity of joint products  $P_i$  produced at the split-off point,  $q_{i0}$  the quantity of joint products  $P_i$  sold at the split-off point (quantity sold of  $P_{i0}$ ),  $q_{i1}$  the quantity of joint products  $P_i$ sold after further processing in the separate processes i (quantity sold of  $P_{i1}$ ),  $e_i$  the production coefficient of  $Q_0$ and quantity produced of joint products  $P_i$  (i.e.,  $q_i$ ),  $a_i$  the production coefficient of joint products  $P_i$  before and after further processing,  $V_{jk}$  the capacity of the kth activity at level j available for production (j = U denotes unit-level; j = B denotes batch-level; j = P denotes process-level),  $C_{jk}$ the unit cost of performing the kth activity at level j,  $X_{ijk}$ the quantity of activity k at level i consumed in the separate processes i for processing one unit of input,  $X_{0ik}$ the quantity of activity k at level j consumed in the common process for processing one unit of input,  $m_i$  the unit cost of raw material in the separate processes i, and  $m_0$  the unit cost of raw material in the common process.

The first and the second terms of Eq. (1) denote total revenues. The third term represents the labor and overhead resources supplied to production, and are treated as period expenses, which are independent of the quantities joint products and further processing products. The fourth and the fifth terms are the material cost of separate processes and joint process, respectively.

Eq. (2) describes the relationship between quantity input and output. The input quantity of the joint process is  $Q_0$  and the output quantities of joint products at the split-off point are  $q_i$  (i = 1,2,3,...,n). The relation between  $Q_0$  and  $q_i$  can be written as follows:

$$q_i = Q_0 \times e_i \tag{4}$$

where  $e_i$  denotes the production coefficient of  $Q_0$  and  $q_i$ . Joint products  $P_i$  can be sold at the split-off point (i.e., product  $P_{i0}$ ) or processed further in a separate process i (i.e., product  $P_{i1}$ ). The relation between  $q_{i1}$  and  $q_i - q_{i0}$  can be written as follows:

$$q_{i1} = (q_i - q_{i0})a_i \text{ or } q_{i1}/a_i = (q_i - q_{i0})$$
 (5)

where  $a_i$  is the production coefficient of  $q_{i1}$  and  $q_i-q_{i0}$  (i.e., production coefficient of joint products  $P_i$  before and after further processing). Eq. (3) limits the quantity of the kth activity used in production.

Theoretically, joint production may occur in fixed or variable proportions. In the former case no substitutability exists between each joint product. The fixed proportions may be determined either by nature or by the technology. Some literature on joint products were confined the fixed proportions case (e.g. Amey, 1984). However, technology also may allow variability in production proportions, within limits, in many joint production operations. The petroleum refining or chemical industries are common examples of this situation. Thus, some authors provided a formal analysis for a linear trade-off ratio between the joint products within the limits of variability (e.g. Hartley, 1971). To simplify the model and focus on the application of TOC and ABC, this study assumes the proportions are fixed. In practice, due to technology constraints, the tradeoff ratio may exist in the form of several possible constants instead of the linear function. Under these circumstances, we could utilize the models in this paper and perform a sensitivity analysis under these possible constant to lead to an optimal solution.

#### 3.2. The ABC model

The model of assessment of the desirability of further processing joint products beyond the split-off point with the revised ABC model, as shown in Fig. 2, may be expressed as

$$\begin{aligned} \text{Max } Z &= \left( \sum_{i=1}^{n} p_{i0} q_{i0} + \sum_{i=1}^{n} p_{i1} q_{i1} \right) - \left( \sum_{i=1}^{n} m_{i} q_{i1} / a_{i} + Q_{0} m_{0} \right) \\ &- \left( \sum_{j \in U} \sum_{i=1}^{n} \sum_{k=1}^{m} C_{jk} X_{ijk} q_{i1} / a_{i} + \sum_{j \in U} \sum_{k=1}^{m} C_{jk} X_{0jk} Q_{0} \right) \\ &- \left( \sum_{j \in B} \sum_{i=1}^{n} \sum_{k=1}^{m} C_{jk} \beta_{ij} B_{ij} + \sum_{j \in B} \sum_{k=1}^{m} C_{jk} \beta_{0j} B_{0j} \right) \\ &- \left( \sum_{j \in P} \sum_{i=1}^{n} \sum_{k=1}^{m} C_{jk} \rho_{ij} R_{ij} + \sum_{j \in P} \sum_{k=1}^{m} C_{jk} \rho_{0j} R_{0j} \right) \end{aligned}$$
(6)

S.T.

(Quantity of input and output constraints):

$$Q_0 = (q_{i0} + q_{i1}/a_i)/e_i, \quad i = 1, 2, \dots, n$$
(7)

(Unit-level activity constraints):

$$\sum_{i=1}^{n} X_{ijk} q_{i1} / a_i + X_{0jk} Q_0 \leqslant Vjk, \quad k = 1, 2, \dots, m; \quad j \in U$$
 (8)

(Batch-level activity constraints):

$$\sum_{i=1}^{n} \beta_{ij} B_{ij} + \beta_{0j} B_{0j} \leqslant Vjk, \quad k = 1, 2, \dots, m; \quad j \in B$$
 (9)

$$q_{i1}/a_i \leq b_{ij}B_{ij}, \quad i = 1, 2, \geq, \geq, \dots, n; \quad j \in B$$
 (10)

$$Q_0 \leqslant b_{0i}B_{0i}, \quad j \in B \tag{11}$$

(Process-level activity constraints):

$$\sum_{i=1}^{n} \rho_{ij} R_i + \rho_{0j} R_0 \leqslant Vjk, \quad k = 1, 2, \dots, m; \quad j \in P$$
 (12)

$$R_0 - R_i \geqslant 0, \quad i = 1, 2, \dots, n$$
 (13)

$$R_0 + R_i + R_2 \geqslant 1, \quad i = 1, 2, ..., n$$
 (14)

 $R_i: 0-1 \text{ variables}, i=1,2,...,n$ 

 $B_{ij}$ : Non-negative integer variables,  $i = 1, 2, ..., n; j \in B$ 

where  $\beta_{0i}$  the requirement of the activity driver of batchlevel activity j ( $j \in B$ ) per batch for the joint process,  $\beta_{ii}$  the requirement of the activity driver of batch-level activity j  $(j \in B)$  per batch for the separate process i,  $B_{0i}$  the number of batches of batch-level activity j  $(j \in B)$  for the joint process,  $B_{ii}$  the number of batches of batch-level activity j $(j \in B)$  for the separate process i,  $b_{0i}$  the number of units per batch of batch-level activity j ( $j \in B$ ) for the joint process,  $b_{ii}$ the number of units per batch of batch-level activity j  $(j \in B)$  for the separate process i,  $\rho_{0i}$  the requirement of the activity driver of process-level activity j ( $j \in P$ ) for the joint process,  $\rho_{ij}$  the requirement of the activity driver of process-level activity for the separate process i, Ro the indicator for producing joint products  $P_i$  ( $R_0 = 1$ ) or not producing joint products  $P_i(R_0 = 0)$  in the joint process,  $R_i$ the indicator for producing products  $P_{i1}$  ( $R_i = 1$ ) or not producing products  $P_{i1}$  ( $R_i = 0$ ) in the separate process i.

Other variables and parameters are as mentioned previously.

Eq. (6) represents the total profit function *Z*. The third to the fifth terms of Eq. (6) represent the unit-level, batch-level and process-level costs of a joint process and separate processes, respectively. This revised ABC model for joint products decision emphasizes the activity costs of processes. Thus, joint costs could be viewed as direct resource costs or indirect resource costs of joint process and could be classified into unit-level, batch-level, or process-level. However, for the purpose of decision making, it is unnecessary to allocate joint costs to joint products.

Eq. (8) is the constraint associated with unit-level activities. For example, we use "labor hours" as the activity driver of the unit-level activity "assembling" because every input in each process needs different labor hours. Eqs. (9)-(11) are the constraints associated with batch-level activities, where Eq. (9) is the capacity constraint for batch-level activity j ( $j \in B$ ). For example, we use "setup hours" as the activity driver of the batchlevel activity "setup" because each process needs different setup hours. In this case,  $V_{ik}$  is the available setup hours;  $\beta_{0i}$  and  $\beta_{ii}$  are the needed setup hours per batch for a joint process and separate process i, respectively;  $B_{0i}$  and  $B_{ii}$  are the number of batches needed for a joint process and separate process i, respectively;  $b_{0i}$  and  $b_{ii}$  are the average number of units in each setup batch for a joint process and separate process I, respectively. In fact, there may be a different number of units in each setup batch for a specific

process. However, we can use the average number of units for the purpose of planning.

Egs. (12)–(14) are the constraints associated with process-level activities. Eq. (12) is the capacity constraint for process-level activity j ( $j \in P$ ). For example, we may use "number of drawings" as the activity driver of the processlevel activity "process design". In this case,  $V_{ik}$  is the available number of drawings for the firm's capacity, and  $\rho_{0i}$  and  $\rho_{ii}$  are the number of drawings needed for a joint process and separate process I, respectively. Eqs. (13) and (14) represent the production sequence of a joint process and separate process i. The companies have to produce joint products first, and then assess the desirability of further processing joint products beyond the split-off point. It is impossible for  $R_0 = 0$  and  $R_1 = 1$ . That is, to process a joint product further, or  $R_1 = 1$ , it must first be process in the joint production process, or  $R_0 = 1$ . In the long run, labor and overhead resources will be controllable by management. Any excess capacity could be redeployed to productive uses elsewhere within the firm, or terminated. Thus, the total costs of activities are dependent on the usages of the activity drivers.

The major difference in joint products mix with the TOC and ABC is how labor and overhead resources are incorporated into the decision process. As indicated in Eq. (6), ABC incorporates the cost of material, labor, and overhead resources used at the production process to evaluate whether joint products should be processed further at the split-off point. This is accomplished using an activity driver to measure the quantity of labor and overhead resources of the kth activity at level j at the production process i (i.e.  $X_{0jk}$  or  $X_{ijk}$ ) used in a product's production. Conversely, the TOC treats labor and overhead as period expenses. Therefore,  $X_{0jk}$ ,  $X_{ijk}$  and the associated cost of labor and overhead resources used in production are irrelevant costs.

Eqs. (1) and (6) may lead to different product mix decisions. These decisions reflect the different assumptions of the TOC and ABC about the relevance of labor and overhead cost to product mix decisions. Many authors (e.g. Bakke and Hellberg, 1991; MacArthur, 1993; Holmen, 1995; Kee and Schmidt, 2000) suggest that the TOC is appropriate for short-term decisions, because of its distinction between variable (for example, materials) and fixed (all other) costs. ABC, however, is appropriate for longer-term decisions because it reflects cause-andeffect relationships, generated over time, among activities, products and costs. Kee and Schmidt (2000) provided a contribution to the body of knowledge by developing a general model that optimizes product mix by simultaneously considering ABC data and the physical attributes of production process. The general model exhibited superior performance when compared to a pure ABC and TOC models for product mix decisions. This difference in performance is due to how the general model considers management's discretionary power over labor and overhead costs of production in their formulation.

To understand how management's discretionary power over labor and overhead resources affects the joint products mix, in Section 3.3, we embed the concept of management's discretionary power over labor and overhead costs of

production by Kee and Schmidt (2000) and the revised ABC model for joint products by Tsai (1996b) to develop the general ABC model for joint products mix.

## 3.3. The general ABC model

Let  $D_{jk}$  and  $N_{jk}$  represent the amount of  $V_{jk}$  subject and not subject to management control, respectively. Furthermore, let  $ID_{jk}$  and  $IN_{jk}$  represent the amounts of  $D_{jk}$  and  $N_{jk}$  that are consumed in production, respectively. The general ABC model for selection of an optimal joint products mix incorporating management's degree of control over labor and overhead may be expressed as

$$\operatorname{Max} Z = \left( \sum_{i=1}^{n} p_{i0} q_{i0} + \sum_{i=1}^{n} p_{i1} q_{i1} \right) \\
- \left( \sum_{i=1}^{n} m_{i} q_{i1} / a_{i} + Q_{0} m_{0} \right) \\
- \left\langle \sum_{j \in U, B, P} \sum_{i=1}^{n} \sum_{k=1}^{m} C_{jk} (N_{ijk} + ID_{ijk}) \right. \\
+ \sum_{j \in U, B, P} \sum_{k=1}^{m} C_{jk} (N_{0jk} + ID_{0jk}) \right\rangle$$
(15)

S.T.

(Quantity of input and output constraints):

$$Q_0 = (q_{i0} + q_{i1}/a_i)/e_i, \quad i = 1, 2, \dots, n$$
(16)

(Unit-level activity constraints):

$$\sum_{i=1}^{n} X_{ijk} q_{i1} / a_i + X_{0jk} Q_0 = I N_{jk} + I D_{jk}$$

$$k = 1, 2, \dots, m; \quad j \in U$$
(17)

$$IN_{ik} \le N_{ik}$$
  $k = 1, 2, ..., m; j \in U$  (18)

$$ID_{jk} \leq D_{jk}$$
  $k = 1, 2, ..., m; j \in U$  (19)

(Batch-level activity constraints):

$$\sum_{i=1}^{n} \beta_{ij} B_{ij} + \beta_{0j} B_{0j} \leq INjk + ID_{jk}$$

$$k = 1, 2, \dots, m; \quad j \in B$$
(20)

$$IN_{jk} \le N_{jk} \quad k = 1, 2, \dots, m; \quad j \in B$$
 (21)

$$ID_{ik} \leq D_{ik} \quad k = 1, 2, \dots, m; \quad j \in B$$
 (22)

$$q_{i1}/a_i \leq b_{ij}B_{ij}, \quad i = 1, 2, \dots, n; \quad j \in B$$
 (23)

$$Q_0 \leqslant b_{0i} B_{0i}, \quad j \in B \tag{24}$$

(Process-level activity constraints):

$$\sum_{i=1}^{n} \rho_{ij} R_i + \rho_{0j} R_0 \leqslant INjk + ID_{jk} \quad k = 1, 2, \dots, m; \quad j \in P \quad (25)$$

$$IN_{jk} \leq N_{jk} \quad k = 1, 2, ..., m; \quad j \in P$$
 (26)

$$ID_{ik} \leq D_{ik} \quad k = 1, 2, \dots, m; \quad j \in P$$
 (27)

$$R_0 - R_i \geqslant 0, \quad i = 1, 2, \dots, n$$
 (28)

$$R_0 + R_i + R_2 \geqslant 1, \quad i = 1, 2, \dots, n$$
 (29)

Ri: 0 - 1 variables, i = 1, 2, ..., n

 $B_{ij}$ : Non-negative integer variables,  $i = 1, 2, ..., n, j \in B$ 

where  $N_{0jk}$  the amount of  $N_{jk}$  for the joint process,  $N_{ijk}$  the amount of  $N_{jk}$  for the separate process i,  $ID_{0jk}$  the amount of  $D_{jk}$  used in production for the joint process,  $ID_{ijk}$  the amount of  $D_{jk}$  used in production for the separate process i, Other variables and parameters are as mentioned previously.

Eq. (15) incorporates the labor and overhead resources that management has no control over,  $N_{jk}$ , and the labor and overhead resources, which management has control that is used in production,  $ID_{jk}$  for unit-level, batch-level and process-level activities. Eqs. (17)–(27) state the relationship between the labor and overhead resources used in production and the amount of these resources that management has no control over and control over that are used in production. That is, the total costs of activities including overhead resources costs, which management has no control (treated as period expenses) and the usages of the activity drivers in which management has control over. Eq. (15) is a more general model for selecting an optimal joint products mix than either the TOC or ABC.

#### 4. An illustrative case

To illustrate the concepts presented in the previous sections, consider the following example. Assume that Company X's production function is such that one unit of material input processed in a joint process (i.e., Process 0) will yield one unit of Product  $P_{10}$  and one unit of Product  $P_{20}$ . Product  $P_{10}$  can be sold at the split-off point for \$60 per unit, or processed further in Process 1 then sold at a price of \$130. One unit of Product  $P_{10}$  processed further in Process 1 will yield one unit of Product  $P_{11}$ . Product  $P_{20}$  can be sold at the split-off point for \$92 per unit, or processed further in Process 2 then sold at a price of \$130. One unit of Product  $P_{20}$  processed further in Process 2 will yield one unit of Product  $P_{21}$ . Fig. 3 shows the production process of Company X.

The data from activity costs of each process and available capacity are presented in Table 1. As indicated

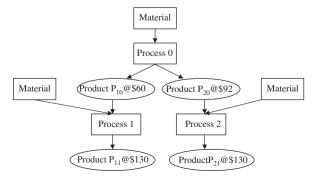


Fig. 3. The production process of Company X.

Table 1
Company X's production processes and activity-based costs

Activity resources	Activity driver		Process				Capacity	Total cost (\$)		Unit cost (\$)
			0	1	2	2				
Labor Overhead	Labor ho	ours	2	1	C	).5	600,000	6,000,0 25,000,		10
Panel B: Detail activity data	for joint	process and sepa	rate proc	esses						
	j	Activity driver	Cost/o	driver		Process 0	Process 1	Process 2	Capacity	Total cost (\$)
Unit-level activity										
Machine A Machine B Machine C	j U	Machine hrs.	\$1.8 2.5 2		$X_{01} \ X_{11} \ X_{21}$	7	2	4	2,500,000 1,000,000 1,500,000	4,500,000 2,500,000 3,000,000
Batch-level activity Setup Units of per batch	В	Setup hrs.	\$30,0	00	$eta_{01,eta_{i1}} \ b_{01,} \ b_{i1}$	3 10,000	1 5000	2 5000	300	9,000,000
Process-level activity Design Total overhead resources cos	P t	Drawings	\$20,0	00	$ ho_{01,} ho_{i1}$	180	80	40	300	6,000,000 25,000,000
Panel C: Cost of joint process	s and sep	parate processes								
			Proces	ss 0			Process	1		Process 2
Unit-level cost Direct material (\$) Direct labor <sup>a</sup> Machine hours <sup>b</sup> Total unit-level cost (\$)			30 20 12.6 62.6				10 10 5 25			6 5 8 19
Batch-level cost Setup (\$) Process-level cost			90,00	0			30,000			60,000
Design (\$)			3,600	,000			1,600,00	00		800,000

<sup>&</sup>lt;sup>a</sup>  $$10 \times 2$ ,  $$10 \times 1$  and  $$10 \times 0.5$ .

in Panel A of Table 1, two activity resources, labor and overhead, are used in production. Processes 0, 1 and 2 require 2, 1 and  $\frac{1}{2}h$  of labor, respectively, for processing one unit of input. The firm has  $600,000\,h$  of production capacity in a year at an expected cost of \$6,000,000. The unit cost of labor per hour is \$10. Processes 0, 1 and 2 require 7, 2 and 4h of machine time, respectively, for processing one unit of input. The expected cost of overhead resources is \$25,000,000. Company X needs the following main activities in producing these joint products and processed further products: one unit-level activities, Machining, one batch-level activities, Setup and one process-level activity, Design. The detail activity data for each process and available capacity are presented in Panel B of Table 1.

The unit cost for the three processes is provided in Panel C of Table 1. Raw material costs were directly traced to production processes based on their usage.

The activity drivers for labor and machining activity are the number of labor hours and machine hours, respectively, used in the processes. Labor and machining costs were then traced to processes based on the number of labor hours and machine hours, respectively, used in the processes. The activity drivers for setup and design activity are the number of setup hours and drawings, respectively, used in the processes. Setup and design costs were then traced to processes based on the number of setup hours and drawings, respectively, used in the processes. As indicated in Panel C of Table 1, unit labor hour costs of Processes 0, 1 and 2 are \$20, \$10 and \$5, respectively, unit machine hour costs of Processes 0, 1 and 2 are \$12.6, \$5 and \$8, respectively, unit setup hour costs of Processes 0, 1 and 2 are \$90,000, \$30,000 and \$60,000, respectively, unit drawing costs of Processes 0, 1 and 2 are \$3,600,000, \$1,600,000 and \$800,000, respectively.

<sup>&</sup>lt;sup>b</sup>  $$1.8 \times 7$ ,  $$2.5 \times 2$  and  $$2 \times 4$ .

#### 4.1. Decision making with TOC and ABC

By using Eqs. (1)–(3), the example is formulated as follows:

$$\max Z = 60q_{10} + 92q_{20} + 130q_{11} + 130q_{21} -30Q_0 - 10q_{11} - 6q_{21} - 31,000,000$$

S.T. 
$$Q_0 = q_{10} + q_{11}$$
  
 $Q_0 = q_{20} + q_{21}$   
 $Q_0 + q_{11} + 0.5q_{21} \le 600,000$   
 $7Q_0 \le 2,500,000; \quad 2q_{11} \le 1,000,000; \quad 4q_{21} \le 1,500,000$   
 $3B_{01} + B_{11} + 2B_{21} \le 300$   
 $Q_0 - 10,000B_{01} \le 0, \quad q_{11} - 5000B_{11} \le 0$   
 $q_{21} - 5000B_{21} \le 0$   
 $180R_0 + 80R_1 + 40R_2 \le 300$   
 $R_0 - R_1 \ge 0, \quad R_0 - R_2 \ge 0, \quad R_0 + R_1 + R_2 \ge 1$ 

where  $Q_0$ ,  $q_{10}$ ,  $q_{11}$ ,  $q_{20}$ ,  $q_{21}$ ,  $B_{02}$ ,  $B_{12}$ ,  $B_{22} \geqslant 0$  and will be an integer.  $R_0$ ,  $R_1$ ,  $R_2$ , are 0–1 variables.

This is an MIP model. We solve this problem and obtain an optimal solution for the TOC model. The optimal solution and profit are presented in Table 2.

In Table 2, information developed from the TOC model, Eq. (1), was solved using the data in Table 1. The firm would manufacture joint products, Products  $P_{10}$  and  $P_{20}$ , with  $P_{10}$  being sold at the split-off point, and Product  $P_{20}$  being processed further in Process 2. This would lead to produce 240,000 units of Product  $P_{10}$  and 240,000 units of Product  $P_{21}$ . According to the profit analysis in Panel C, the firm would obtain a net profit \$5,960,000.

Similarly, by using Eqs. (6)–(14), the example is formulated as follows and the optimal solution and profit are presented in Table 3.

**Table 2**Company X's TOC solutions and profit analysis

Panel A: Quantity input in production process					
Process 0		Process 1	Process 2		
240,000		0	240,000		
Panel B: Quanti	ty of products sol	d			
Product P <sub>10</sub>	Product P <sub>20</sub>	Product P <sub>11</sub>	Product P <sub>21</sub>		
240,000	0	0	240,000		
Panel C: Profit					
		Total (\$)			
Total revenue		45,600,000			
Material cost		8,640,000			
Labor cost		6,000,000			
Overhead cost		25,000,000			
Net profit		5,960,000			

**Table 3**Company X's ABC solutions and profit analysis

Panel A: Quant	ity input in producti	on process	
Process 0	Process	Process 2	
300,000	0		0
Panel B: Quanti	ty of products sold		
Product P <sub>10</sub>	Product P <sub>20</sub>	Product P <sub>11</sub>	Product P <sub>21</sub>
300,000	300,000	0	0
Panel C: Profit			
		Total (\$)	
Total revenue Material cost Labor cost Overhead cost		45,600,000 9,000,000 6,000,000	
Machine cost Setup cost Design cost		3,780,000 2,700,000 3,600,000	
Net profit		20,520,000	

$$\begin{aligned} \text{Max } Z &= (60q_{10} + 92q_{20} + 130q_{11} + 130q_{21}) \\ &- (30Q_0 + 10q_{11} + 6q_{21}) \\ &- (20Q_0 + 10q_{11} + 5q_{21}) \\ &- (12.6Q_0 + 5q_{11} + 8q_{21}) \\ &- (90,000B_{01} + 30,000B_{11} + 60,000B_{21}) \\ &- (3,600,000R_0 + 1,600,000R_1 + 800,000R_2) \end{aligned}$$

S.T.

(Quantity of input and output constraints)

$$Q_0 = q_{10} + q_{11}, \quad Q_0 = q_{20} + q_{21}$$
(Unit-level direct labor constraints)

$$2Q_0 + q_{11} + 0.5q_{21} \leq 600,000$$

(Machine capacity constraints)

$$7Q_0 \le 2,500,000$$
,  $2q_{11} \le 1,000,000$ ,  $4q_{21} \le 1,500,000$ 

(Batch-level activity constraints)

$$3B_{01} + B_{11} + 2B_{21} \leq 300$$

$$Q_0 - 10,000B_{01} \leqslant 0, \quad q_{11} - 5000B_{11} \leqslant 0,$$
  
 $q_{21} - 5000B_{21} \leqslant 0$ 

(Process-level activity constraints)

$$180R_0 + 80R_1 + 40R_2 \le 300$$
  
 $R_0 - R_1 \ge 0$ ,  $R_0 - R_2 \ge 0$ ,  $R_0 + R_1 + R_2 \ge 1$ 

where  $Q_0$ ,  $q_{10}$ ,  $q_{11}$ ,  $q_{20}$ ,  $q_{21}$ ,  $B_{02}$ ,  $B_{12}$ ,  $B_{22} \ge 0$  and will be an integer.  $R_0$ ,  $R_1$ ,  $R_2$ , are 0–1 variables.

In Table 3, information developed from the ABC model, Eq. (6), was solved using the data in Table 1. The firm would manufacture joint products and sold at the split-off point. This would lead to produce 300,000 units of Product  $P_{10}$  and

300,000 units of Product  $P_{20}$ . According to the profit analysis in Panel C, the firm would obtain a net profit \$20,520,000.

As indicated in Tables 2 and 3, there are substantial net profit differences under the ABC model and the TOC model. This means that, if overhead resources are discretionary and unused overhead activity resources can be redeployed to other uses within the firm or terminated, then the ABC model will obtain the highest profit in the long-term optimal capacity structure. Conversely, if overhead activity resources are non-discretionary, the TOC model will obtain the highest profit in the short-term optimal capacity structure.

## 4.2. Decision making with the general ABC model

In practice, management has neither complete control nor total lack of control over labor and overhead resources. To incorporate management's degree of control over resources into decision processes, assume that half of the resources of machining function and setup function are discretionary and that the remainders are non-discretionary.

By using Eqs. (15)–(29), the example is formulated as follows and the optimal solution and profit are presented in Table 4.

$$\begin{aligned} \text{Max } Z &= (60q_{10} + 92q_{20} + 130q_{11} + 130q_{21}) \\ &- (30Q_0 + 10q_{11} + 6q_{21}) \\ &- (20Q_0 + 10q_{11} + 5q_{21}) \\ &- 1.8(1,250,000 + ID_{0u1}) \\ &- 2.5(500,000 + ID_{1u1}) \\ &- 2(750,000 + ID_{2u1}) \\ &- 30,000(150 + 3ID_{0B1}) \\ &+ ID_{1B1} + 2ID_{2B1}) - 6,000,000 \end{aligned}$$

Table 4
Company X's general ABC model solutions and profit analysis

Panel A: Quantity	input in produ	iction process			
Process 0	Proc	Process 2			
178,571	165,	155,000			
Panel B: Quantity of products sold					
Product P <sub>10</sub>	Product P <sub>20</sub>	Product P <sub>11</sub>	Product P <sub>21</sub>		
13,213	23,571	165,358	155,000		
Panel C: Profit					
		Total (\$)			
Total revenue Material cost Labor cost Overhead cost Machine cost Setup cost Design cost		44,607,852 7,940,710 6,000,000 5,000,000 4,500,000 6,000,000			
Net profit		15,167,142			

S.T. (Quantity of input and output constraints)

$$Q_0 = q_{10} + q_{11}, \quad Q_0 = q_{20} + q_{21}$$

(Unit-level direct labor constraints)

$$2Q_0 + q_{11} + 0.5q_{21} \leq 600,000$$

(Machine capacity constraints)

$$7Q_0 \le IN_{0u1} + ID_{0u1}, \ IN_{0u1} \le 1,250,000, \ ID_{0u1} \le 1,250,000$$
  
 $2q_{11} \le IN_{1u1} + ID_{1u1}, \ IN_{1u1} \le 500,000, \ ID_{1u1} \le 500,000$   
 $4q_{21} \le IN_{2u1} + ID_{2u1}, \ IN_{2u1} \le 750,000, \ ID_{2u1} \ 1 \le 750,000$ 

(Batch-level activity constraints)

$$\begin{aligned} 3B_{02} + B_{12} + 2B_{22} &= 3(IN_{0B1} + ID_{0B1}) \\ + (IN_{1B1} + ID_{1B1}) + 2(IN_{2B1} + ID_{2B1}) \\ 3IN_{0B1} + IN_{1B1} + 2IN_{2B1} \leqslant 150, \\ 3ID_{0B1} + ID_{1B1} + 2ID_{2B1} \leqslant 150 \\ Q_0 - 10,000(IN_{0B1} + ID_{0B1}) \leqslant 0, \\ q_{11} - 5000(IN_{1B1} + ID_{1B1}) \leqslant 0, \\ q_{21} - 5000(IN_{2B1} + ID_{2B1}) \leqslant 0 \end{aligned}$$

where  $Q_0$ ,  $q_{10}$ , q = 1, 2, ..., n,  $q_{11}$ ,  $q_{20}$ ,  $q_{21}$ ,  $B_{02}$ ,  $B_{12}$ ,  $B_{22} \ge 0$  and will be an integer.

In Table 4, information developed from the general ABC model, Eq. (15), was solved using the data in Table 1 with the discretionary and non-discretionary overhead resources to determine an optimal joint products further processing decision. As indicated in Panels A and B of Table 4, the firm would manufacture joint products, 178,571 units of Product  $P_{10}$  and 178,571 units of Product 13 units of Product  $P_{10}$  and 23,751 units of Product  $P_{20}$  being sold at the split-off point. Then, 165,358 units of Product  $P_{10}$  being processed further in Process 1 and 155,000 units of Product  $P_{20}$  being processed further in Product  $P_{10}$ , 23,751 units of Product  $P_{20}$ , 165,358 units of Product  $P_{11}$  and 155,000 units of Product  $P_{21}$ . According to the profit analysis in Panel C, the firm would obtain a net profit \$15,167,142.

When a firm's management has neither completed nor zero control over overhead resources, the TOC and ABC may lead to suboptimal decisions. A comparison of the joint products mix with TOC, ABC and the general ABC model is provided in Table 5. When the firm cannot avoid the unused overhead resource costs, the TOC model obtains the highest profit (income based on the cost of resources supplied to production). This is the best profit in the short run. On the contrary, the ABC model obtains the highest profit (income based on resources used in production) because the firm can completely avoid the unused overhead resource costs. With no capacity expansions, this is the highest profit potential in the long run. In a general situation, the firm only can partly avoid the unused overhead resource costs. Accordingly, the general ABC model can generate the highest profit (income less non-discretionary capacity cost) and lead to the optimal joint products mix in most practical environments.

**Table 5**Company X's comparative analysis of the three decision models

	TOC	ABC	General ABC
Product P <sub>10</sub>	240,000	300,000	13,213
Product P <sub>20</sub>	0	300,000	23,571
Product P <sub>11</sub>	0	0	165,358
Product P <sub>21</sub>	240,000	0	155,000
Panel B: Resources consumed			
Resources used in production			
Labor hours	600,000	600,000	600,000
Non-discretionary machine hours	2,000,000	1,250,000	2,200,713
Discretionary machine hours	640,000	850,000	0
Non-discretionary setup hours	150	90	150
Discretionary set hours	18	0	0
Non-discretionary drawings	220	180	300
Unused resources			
Non-discretionary machine hours	500,000	1,250,000	299,287
Discretionary machine hours	1,860,000	1,650,000	2,500,000
Non-discretionary setup hours	0	60	0
Discretionary set hours	132	150	150
Non-discretionary drawings	80	120	0
Panel C: Profit			
Revenue (\$)	45,600,000	45,600,000	44,607,852
Cost of resources used in production			
Material cost	8,640,000	9,000,000	7,940,710
Labor cost	6,000,000	6,000,000	6,000,000
Machine cost	4,944,000	3,780,000	4,316,785
Setup cost	5,040,000	2,700,000	4,500,000
Design cost	4,400,000	3,600,000	6,000,000
Income based on resources used in production (\$)	16,576,000	20,520,000	15,850,357
Unused non-discretionary capacity cost	2,850,000	6,950,000	683,215
Income less non-discretionary capacity cost (\$)	13,726,000	13,570,000	15,167,142
Unused discretionary capacity cost	7,766,000	7,970,000	9,500,000
Income based on the cost of resources supplied to production (\$)	5,960,000	5,600,000	5,667,142

## 5. Conclusion

In order to maximize total profit under the various constraints of resources, the firms who produce joint products have to assess the desirability of further processing joint products after the split-off point. Three decision models for joint products mix are presented in this paper and a simplified case is used to demonstrate the process of decision making and profit analysis under each model.

To achieve the short-term decision, TOC assumes that a firm's management has no control over its labor and overhead resources and views all fixed cost allocations as irrelevant and only assigns truly variable costs to products. The joint products mix with TOC involves maximizing throughput subject to the firm's bottleneck activities used in the production processes and will lead to the highest short-term profit. Conversely, to achieve the long-tern decision, the ABC model for joint products is

used to trace resource costs to processes and then to products. Consequently, it improves the accuracy of processes cost data derived from the traditional volume or unit-based costing systems. In the long run, when a firm's management has complete control over its labor and overhead resources, the ABC model will obtain the highest long-term profit.

However, in general circumstances, a firm's management has varying degrees of control over its labor and overhead resources. If excess capacity cannot be redeployed or terminated, the joint products mix with the ABC model may overstate its profitability and if excess capacity could be redeployed to productive uses elsewhere within the firm or terminated, the joint products mix with the TOC model may understate its profitability. An ABC model embedding management's discretionary power over labor and overhead resources was developed and used to make joint product mix decisions. As illustrated in the paper, an ABC model embedding management discretionary power

over labor and overhead resources lead to an optimal joint product mix decision when the ABC and TOC model did not

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