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An Innovative Method for the Analysis of Scalar Modes of 3-D Dielectric Optical Waveguides

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Abstract

In this paper, an exact Fourier cosine method is proposed for the numerical calculation of the propagation constant in the rib-type dielectric waveguides with arbitrary index profile. Example is given for rib waveguides based on semiconductor material and silicon dioxide. It establishes the accuracy of Fourier cosine method as a criterion in calculation of critical design parameters such as propagation constant and mode field profile. Moreover, the accuracy and CPU time of this method are presented as well as compared with the commercial software Beam PROP.

Key words: Fourier cosine series, propagation constant, and index profiles, Newton-Raphson.

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以一新穎方法分析三維光波導之純量模態

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摘 要

本論文提出以一正確傅立葉餘弦方法在任意折射率分佈之脊形介電光波導之純量模態的數值計算，並以半導體材料二氧化矽製成波導佐證，它建立了以本方法所計算之光傳播常數及場形分佈為設計光纖元件之標準。

關鍵詞：傅立葉餘弦級數、傳播常數、折射率分佈

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I. Introduction

With the great progress of integrated optic circuit, single mode waveguide is important to be used as a fundamental device. Therefore, numerical methods to analyze waveguides are essential to understand propagation characteristics of light and to develop the optical design of the optical devices. These methods consist of the finite difference method (FDM) [1-2], the finite element method FEM) [3-4] and the beam propagation method (BPM) [5], the generalized Fourier variational method [6] or the Fourier operator transform method [7]. Recently, we proposed a new method suitable to waveguide analysis and demonstrated the efficiency and the accuracy of the method by comparison with the commercial software Beam PROP.

In this paper, we propose an exact semi-analytic approach to find the modal indices and modal fields of optical rib waveguides. In the approach, the cross section of a rib waveguide is divided into several regions, in each of which both the refractive index profile and the field distribution is expressed as Fourier cosine series, respectively. In each region, a solution form of modal fields can be derived from a second-order differential matrix equation. The accuracy of finding the modal index depends on the number of terms used in expanding the aforementioned refractive profile (as well as the modal field) into Fourier cosine series. The method of expanding both refractive index profile and the field distribution into Fourier cosine series has proven to be relatively efficient and accurate in finding the modes of one-dimensional optical waveguides (i.e., slab waveguides). This technique has been for the first time applied to optical rib waveguides here. In the proposed method, the field in each region is expressed into a Fourier cosine series. Note that neither an effective rib waveguide nor mode expansion (for the rib region) is used by the proposed method. Furthermore, the index profile is expressed into a Fourier cosine series in each region and then substituted in the wave equation to obtain a corresponding matrix equation, from which an exact closed-form solution for the field can be found. In the following one can find that the proposed method provides much better accuracy in determining the modal index than these aforementioned semi-analytic methods.

In Section two, the theory of the proposed matrix method for the cases of scalar modes is outlined. In Section three, we present the computational results, where one can see the accuracy and efficiency the method provides. Finally, Section four followed by a brief summary for this paper.

II. Theory

In the analysis of the scalar mode of the rib-type waveguide, the cross-section of the considered rib waveguide is divided into four regions, as shown in Fig1. The coordinates y1 to y3 represent, respectively, the boundaries between two corresponding adjacent regions. The coordinates x1 and x2 are the positions of the two rib's sidewalls. Clearly, for regions I (i.e., for $0<y<y1$), II ($y1<y<y2$), and IV ($y3<y<y4$), the refractive indices are n_2, n_1 and n_3 (all of which are constant), respectively; while for region III (i.e., for $y2<y<y3$), the refractive index $n_0^2(x)$ follows the distribution

$$\begin{aligned} n_0^2(x) &= n_1^2, \ \textit{for } x_1 < x < x_2 \\ &= n_3^2, \ \textit{for } 0 < x < x_1 \\ &\qquad \textit{and } \ x_2 < x < x_3 \end{aligned} \tag{1}$$

Note that the function $n_0^{\ 2}(x)$ can be extended into an even periodic function with a period of $2\cdot x_3$. Then we can have the Fourier cosine series expansion $n_0^{\ 2}(x) = \sum_{n=0}^{N} a_n \cos n\Delta w x$ with $\Delta w = p / x3$ and N being large enough. In each region defined in Fig. 1, the electric field distribution can be likewise expanded, i.e., we can write the field in region i as $e_i\ (x,y) = \sum_{n=0}^{N} e^i{}_n(y) \cos n\Delta w x$, a periodic distribution extended over $-\infty < x < \infty$. Here $e_n^{\ i}$ represents the amplitude of a spatial spectral component in the Fourier expansion for region i. Now we consider the scalar wave equation

$$\frac{\partial^2 \varepsilon}{\partial y^2} + \frac{\partial^2 \varepsilon}{\partial x^2} + (k_0^{\ 2} n^2(x,y) - \beta^2)\varepsilon \ = 0 \tag{2}$$

separately for each region, but noting that $n^2(x,y)$ is equal to $n_2^{\ 2}$, $n_1^{\ 2}$, $n_0^{\ 2}(x)$, and $n_3^{\ 2}$ for regions I, II, III and IV, respectively. Here in Eq. (2), b is the propagation constant and k_0 the wave number of the free space. For region I, we replace $n^2(x,y)$ of Eq. (2) by $n_2^{\ 2}$ and substitute the Fourier cosine series expansion of the field in the equation. Then we can obtain a series of harmonic terms, i.e., the terms with $\cos m\Delta w x$ (m=0,1,2,...,N), on the left-hand side of Eq. (2). After equating all the coefficients of the cosine terms to zero, we have a set of differential equations that can be expressed in the following matrix form.

$$\frac{\partial^2 E_1}{\partial y^2} + (k_0{}^2 n_2{}^2 \mathrm{I} - W - \beta^2 \mathrm{I}) E_1 = 0 \qquad (3)$$

Here the vector E_1 is defined by $E_1 = [e_0{}^1, e_1{}^1, e_2{}^1, \cdots, e_N{}^1]^T$ with T representing the transpose; I is the identity matrix and W is a diagonal matrix defined as

$$W = \begin{bmatrix} 0 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & (\Delta w)^2 & 0 & \cdot & \cdot & \cdot \\ 0 & 0 & (2\Delta w)^2 & \cdot & \cdot & \cdot \\ \cdot & \cdot & 0 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & (N-1)\Delta w)^2 & 0 \\ 0 & \cdot & \cdot & \cdot & 0 & (N\Delta w)^2 \end{bmatrix} \qquad (4)$$

In a similar way we can obtain two differential matrix equations, respectively, for regions II and IV, and they are

$$\frac{\partial^2 E_2}{\partial y^2} + (k_0{}^2 n_1{}^2 \mathrm{I} - W - \beta^2 \mathrm{I}) E_2 = 0 \qquad (5)$$

and

$$\frac{\partial^2 E_4}{\partial y^2} + (k_0{}^2 n_3{}^2 \mathrm{I} - W - \beta^2 \mathrm{I}) E_4 = 0 \qquad (6)$$

where vectors E_2 and E_4 are given as $E_2 = [e_0{}^2, e_1{}^2, e_2{}^2, \cdots, e_N{}^2]^T$ and $E_4 = [e_0{}^4, e_1{}^4, e_2{}^4, \cdots, e_N{}^4]^T$, respectively.

For region III, we assume $n^2(x,y) = \sum_{n=0}^{N} a_n \cos n\Delta w x$ and the wave field is $\sum_{n=0}^{N} e^3{}_n(y) \cos n\Delta w x$ then we substitute these two terms into Eq. (2). A manipulation similar to that leading to Eq. (3) (to Eqs. (5) and (6) as well) then gives the matrix equation

$$\frac{\partial^2 E_3}{\partial y^2} + (k_0{}^2 A - W - \beta^2 I) E_3 = 0 \qquad (7)$$

where matrix A is defined as

$$A = \frac{k_0{}^2}{2} \begin{bmatrix} 2a_0 & a_1 & a_2 & a_3 & \cdots & a_{N-1} & a_N \\ 2a_1 & 2a_0 + a_2 & a_1 + a_3 & a_2 + a_4 & \cdots & a_{N-2} + a_N & a_{N-1} + a_{N+1} \\ 2a_2 & a_1 + a_3 & 2a_0 + a_4 & a_1 + a_5 & \cdots & a_{N-3} + a_{N+1} & a_{N-2} + a_{N+2} \\ 2a_3 & a_2 + a_4 & a_1 + a_5 & 2a_2 + a_6 & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdots & 2a_0 + a_{2(N-1)} & \cdot \\ 2a_n & a_{N-1} + a_{N+1} & \cdot & \cdot & \cdots & \cdot & 2a_0 + a_{2N} \end{bmatrix}$$

(8)

Explicit solutions to Eqs. (3), (5), (6) and (7) are found, respectively, to be

$$E_1(y) = \begin{bmatrix} g_0 \exp[\sqrt{\beta^2 - \kappa_0}(y - y_1)] \\ g_1 \exp[\sqrt{\beta^2 - \kappa_1}(y - y_1)] \\ g_2 \exp[\sqrt{\beta^2 - \kappa_2}(y - y_1)] \\ \cdot \\ \cdot \\ \cdot \\ g_N \exp[\sqrt{\beta^2 - \kappa_N}(y - y_1)] \end{bmatrix}$$

$$E_2(y)=\begin{bmatrix} b_0\cos[\sqrt{\gamma_0-\beta^2}(y-y_1)-\phi_0] \\ b_1\cos[\sqrt{\gamma_1-\beta^2}(y-y_1)-\phi_1] \\ b_2\cos[\sqrt{\gamma_2-\beta^2}(y-y_1)-\phi_2] \\ . \\ . \\ . \\ b_N\cos[\sqrt{\gamma_N-\beta^2}(y-y_1)-\phi_N] \end{bmatrix}$$

$$E_4(y)=\begin{bmatrix} d_0\exp[-\sqrt{\beta^2-\delta_0}(y-y_3)] \\ d_1\exp[-\sqrt{\beta^2-\delta_1}(y-y_3)] \\ d_2\exp[-\sqrt{\beta^2-\delta_2}(y-y_3)] \\ . \\ . \\ . \\ d_N\exp[-\sqrt{\beta^2-\delta_N}(y-y_3)] \end{bmatrix}$$

, and

$$E_3 = \sum_{n=0}^{N} Y_n c_n \cos[\sqrt{\lambda_n-\beta^2}(y-y_2)-\theta_n) \quad (9)$$

Here k$_i$, g$_i$, l$_i$ and d$_i$ $(i=0,1,2,\cdots,N)$ are the eigenvalues of matrices $(k_0{}^2 n_2{}^2 I-W)$, $(k_0{}^2 n_1{}^2 I-W)$, $(k_0{}^2 A-W)$ and $(k_0{}^2 n_3{}^2 I-W)$, respectively; and $Y_i (i=0,1,2,\cdots,N)$ is the eigenvector of matrix $(k_0{}^2 A-W)$. In the identities of (9), the parameters g_i, b_i, c_i, d_i, f$_i$ and q$_i$ $(i=0,1,2,\cdots,N)$ should satisfy the boundary conditions at the interfaces between two adjacent regions (e.g. regions I and II; regions II and III; regions III and IV). The boundary conditions here state that the field and its first derivative with respect to y are continuous at positions y1, y2, and y3. Consequently, we have the following boundary conditions:

$$E_1(y_1)=E_2(y_1)\ ,\ \frac{\partial E_1}{\partial y}\Big|y_1=\frac{\partial E_2}{\partial y}\Big|y_1$$

$$E_2(y_2)=E_3(y_2)\ ,\ \frac{\partial E_2}{\partial y}\Big|y_2=\frac{\partial E_3}{\partial y}\Big|y_2$$

$$E_3(y_3)=E_4(y_3)\ ,\ \frac{\partial E_3}{\partial y}\Big|y_3=\frac{\partial E_4}{\partial y}\Big|y_3 \quad (10)$$

Using the six boundary conditions above, we can derive two matrix identities as shown below.

$$K(\beta)\cdot X-L(\beta)\cdot Y=0 \quad (11)$$

$$M(\beta)\cdot X+N(\beta)\cdot Y=0 \quad (12)$$

Here vectors X and Y are defined as X= $[c_0\cos\mathrm{q}_0, c_1\cos\mathrm{q}_{1,}.....c_N\cos\mathrm{q}_N]^T$ and Y= $[c_0\sin\mathrm{q}_0, c_1\sin\mathrm{q}_{1,}.....c_N\sin\mathrm{q}_N]^T$ Matrices K, L, M and N contain elements that depend on b .

To solve for b , we rewrite Eqs. (11) and (12) as

$$\begin{bmatrix} K(\beta) & -L(\beta) \\ M(\beta) & N(\beta) \end{bmatrix}\cdot\begin{bmatrix} X \\ Y \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (13)$$

Since there should exist a nontrivial solution for Eq. (13), the following equation must hold.

$$\det\left\{\begin{bmatrix} K(\beta) & -L(\beta) \\ M(\beta) & N(\beta) \end{bmatrix}\right\} = 0 \qquad (14)$$

where det{.} represents the determinant of a matrix. The equation in (14) is thus used to determine the modal index (which is equal to β / k_0). A Newton-Raphson algorithm [8] is quite efficient in solving Eq. (14) and is used in this study.

III. Numerical Results

Here we present numerical results for an example of rib waveguides. The example, denoted by structure 1 in Table 1, is a waveguide based on semiconductor material and silicon dioxide. The refractive indices and structural parameters are given in the table for this structure. The single-mode waveguide in structure 1 was studied by Ref [4]. The modal indices of this waveguide calculated by Ref[4] is also given here for a comparison with the numerical results obtained using the commercial software BeamPROP and the proposed method.We ran the software Beam-PROP, and the proposed method in a PC equipped with Pentium-1.25GHz.

Table 2 shows the effective index n_{eff} calculated by the proposed method, the commercial software BeamPROP, and the method of Ref. [4], for the scalar mode of the rib waveguide of structure 1 (single-mode waveguide). In the proposed method, the number of terms in the Fourier cosine series expansion (i.e., N) varies from 15 to 54, and these n_{eff} converge to 3.391147. It is evident that the calculated n_{eff} is almost the same as those obtained by BeamPROP and Ref. [4]. The CPU time spent by the proposed method is less than 1.4 sec for N = 54, while BeamPROP needs much more time.

Fig. 2 and Fig. 3 show the normalized field distribution for the fundamental scalar mode in the x and y direction where the continuities of Ex across the air-dielectric interface along x and y direction are found in those two figures.

IV. Conclusion

In conclusion, we have presented a practical and versatile method for the calculation of propagation field through rib-type structure. As data shown in table2, the proposed method can rapidly calculate the mode index with high accuracy and is more versatile than many existing numerical methods. Consequently, the proposed method would be capable of yield accurate results for waveguides with large refractive index discontinuities over the transverse cross section. Therefore, it may be applied to the simulation and calculation of the polarized waves in strongly guiding structure.

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Table 1: The wavelength and the structural parameters of rib waveguide considered in this paper.

structure	λ	n1	n2	n3	w(μm)	h(μm)	d(μm)	x1(μm)	x3(μm)	y1(μm)	y4(μm)
1	1.55 μm	3.44	3.34	1	2	1.3	0.2	2.2	6.4	2.2	6.4

Table 2: Comparison of modal indices calculated by the proposed method, the commercial software Beam PROP and the method of Ref. [4], for the scalar mode of the rib waveguide of structure 1.

The proposed method			Beam PROP			Ref.[4]
N	n_{eff}	cpu(sec)	grid sizes (μm)	n_{eff}	cpu(sec)	n_{eff}
15	3.403138	0.15	$\triangle x=\triangle y=0.05,=\triangle z=1$	3.390867	0.3	3.391148
25	3.391911	0.30	$\triangle x=\triangle y=0.025,=\triangle z=1$	3.391070	1.4	
35	3.391402	0.59	$\triangle x=\triangle y=0.01,\triangle z=1$	3.391131	9.8	
40	3.391336	0.71	$\triangle x=\triangle y=0.005,\triangle z=1$	3.391140	90	
46	3.391287	1.03				
50	3.391250	1.22				
54	3.391147	1.39				

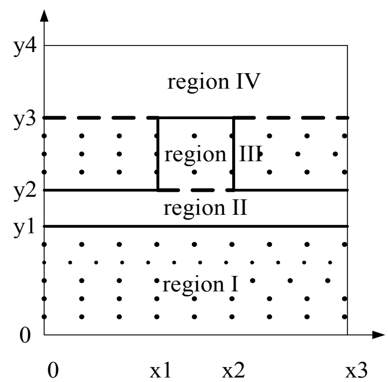

Fig.1 Dividing the cross-section of a rib waveguide into several regions in the cases of scalar modes. y1, y2 and y3 represent the coordinates in the y axis, denoting the interface between regions.

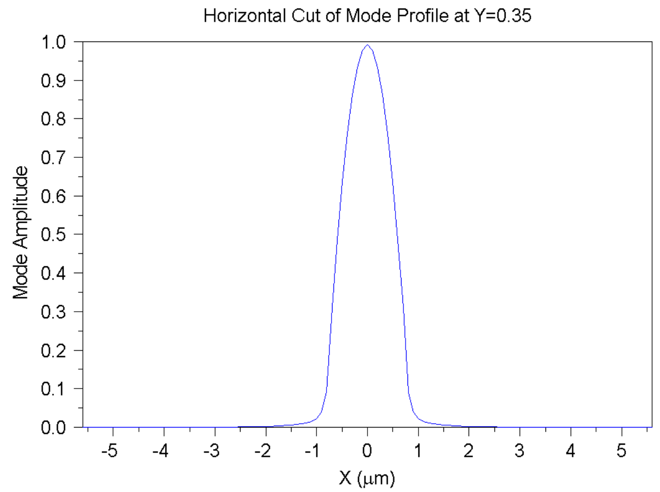

Fig. 2 Normalized field distribution for the fundamental scalar mode (Ex) of structure 1 in the x direction where contour patterns become maximum

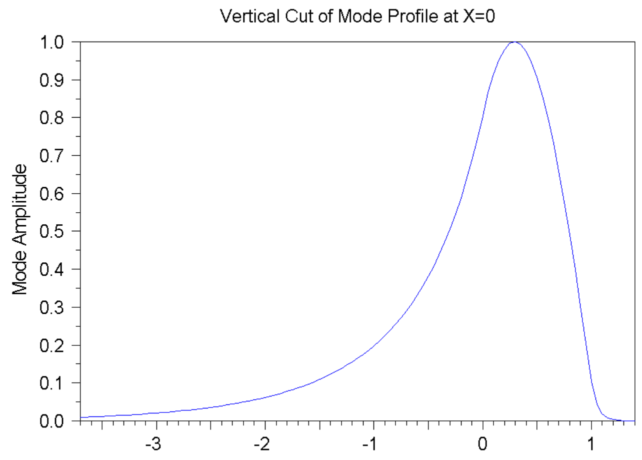

Fig. 3 Normalized field distribution for the fundamental scalar mode (Ex) of structure 1 in the y direction where contour patterns become maximum