以一新穎方法研究多模干涉耦合器極化橫向電波模的傳導特性

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摘 要

本論文中我們提供一新穎方法研究具有任意折射率分佈之縱向結構不變光波導中之波的特性。我們之方法裡，保留了橫向波場對傳播方向之二階導數，它在很多情況下，於近軸近似常被忽略.我們因而導出沿著光波導之任何縱向位置下之簡明數學式子，因而不必使用光束傳導演算技巧。本研究論文已證明我們方法和商業軟體以大角度pade`之光波傳導演算法有相同結果。

關鍵詞：傅立葉餘弦級數，矩陣方法，折射率分佈，光波導。

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A Novel Approach to Study Wave Characteristics of Multimode Interference Couple: Polarized TE Modes

Chin-Sung Hsiao

Abstract

In this paper, we propose a new method to study wave propagation in Longitudinally invariant waveguides with arbitrary index profile. In our method, we keep, in the wave equation, the second order derivative of transverse wave field with respect to the propagation direction, which is usually neglected in paraxial approximation utilized in many cases. An explicit expression for the wave field at any longitudinal position along an optical waveguide is derived, thus excluding the use of beam propagation algorithm for computation. This study has demonstrated that our approach yields the same results as those by using a commercial software in which a beam propagation method with Pade' approximation is used.

Keywords: Fourier cosine series, matrix method, index profile, optical waveguides.

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I.Introduction

Beam- propagation methods (BPM) have been frequently used to compute the wave field along an optical waveguide. With these methods, the field distribution at any longitudinal position of a waveguide can be found by either finite- difference [1-4] or finite-element [5-8] discretization in the transverse domain. These conventional methods, especially the finite-difference BPM algorithm, are nowadays highly referenced and used in commercial softwares [e.g. BPM-CAD and BeamPROP].

In dealing with the wave propagation with the paraxial approximation valid in use, the second order derivative with respect to propagation distance in the wave equation is simply neglected. This then results in a set of first-order linear differential equations, which can be easily solved. If the paraxial approximation is invalid, as is true in many cases, the aforementioned second order derivative should remain in order to obtain exact and accurate solutions in solving the corresponding wave propagation problems.

A recurrence formula for the BPM scheme has been widely employed to consider the effect of the aforementioned second-second derivative in the so-called wide-angle approximation [9,10] . The Pade' approximant is commonly used as one kind of such approximation. The numerical results obtained by using the Pade' approximation in a BPM method are more accurate and closer to exact ones when a higher-order Pade' approximant operator is used [11, 12].

II. The Proposed method

We assume that the transverse domain of the wave field is one-dimensional. That is, we deal with the following wave equation in our problem

$$\partial^2 \omega(x,z) / \partial z^2 + \partial^2 \omega(x,z) / \partial x^2 + k_0 n^2(x) \omega(x,z) = 0 \tag{1}$$

The resultant wave equation in the example of slab waveguides with longitudinally invariant structures corresponds to the following matrix equation:

$$\frac{d^2E}{dz^2} - 2j\beta\frac{dE}{dz} + BE = 0 \qquad (2)$$

Here we have expressed the electric field as

$$\varepsilon\ (x,z) = \sum_{n=0}^{N} e_n(z)\cos\frac{2\pi n}{T}x\ .\exp(-\ j\ \ \beta\ \ z)\ ,$$

where N is a large enough number, T the window (or the period) for Fourier cosine series, and β the propagation constant. In deriving Eq. (2), we have assumed the wave propagates in the z direction and x is the one-dimensional transverse coordinate. The matrix B in Eq. (2) is a constant full matrix, and E is defined as ($E = [e_0, e_1, e_2, \ldots\ldots, e_N]^t$), where *t* represents transpose. In the conventional BPM (beam propagation method) method for solving Eq.(1) with Pade' approximation used, a recurrence formula is employed to obtain approximate results. Basically Eq. (2) can be solved with similar approximation used.

However, such Pade' approximation in dealing with matrix arithmetic could result in quite complicated computing algorithm and yield computational inefficiency. Here we propose a novel method to solve the second -order matrix equation (i.e., Eq. (2)). Since the matrix B in Eq. (2) is not diagonal, it is difficult to solve it directly. In this new method, Eq. (2) is first transformed into a matrix equation such as

$$\left(\frac{d^2F}{dz^2} - 2j\beta\frac{dF}{dz} + \Lambda F = 0 \right) \qquad (3)$$

Here the vector *F* is defined as *E*=*Y* · *F*, where *Y* is a matrix containing all the eigenvectors of B; is the diagonal matrix with its diagonal elements being the eigenvalues of B (see Eq. (5) below).

Once it is solved, the vector E can be obtained. Eq. (3) can now be readily solved because is diagonal. To show how to solve it, we first note that the matrix equation in (3) corresponds to a set of second-order ordinary differential equations:

$$\frac{d^2f_i}{dz^2} - 2j\beta\frac{df_i}{dz} + \lambda_i f_i = 0 \qquad (4)$$

where f_i and λ_i (i=0,1,2,....N) are elements of F and Λ ,respectively. That is, F and Λ are defined here as $F=[f_0, f_1, f_2, \ldots, f_N]^t$ and

$$\Lambda = \begin{bmatrix} \lambda_0 & & & & & \\ & \lambda_1 & & & 0 & \\ & & \lambda_2 & & & \\ & & & \bullet & & \\ & 0 & & & \bullet & \\ & & & & & \lambda_N \end{bmatrix} \tag{5}$$

The solution of Eq.(4) is simply c_{i1} exp $[(j\ \beta+\sqrt{-\beta^2-\lambda_i}\ )\ z\ ]$ + c_{i2} exp $[(j\ \beta-\sqrt{-\beta^2-\lambda_i}\ )\ z\ ]$, where c_{i1} and c_{i2} depend on the initial conditions. This solution is explicit in expression and henceforth the vector E can be explicitly obtained as

$$\mathrm{E}(z)=\sum_{i=0}^{N} P_i\cdot\{a_i \exp\ [(j\,\beta+\sqrt{-\beta^2-\lambda_i}\ )\,z\ ]$$
$$+b_i\ \exp\ [(j\,\beta-\sqrt{-\beta^2-\lambda_i}\,)\ z\ ]\} \tag{6}$$

where a_i and b_i are constants and P_i are the eigenvector of B.

As we can see above, the conventional beam propagation method would not be used to solve the wave propagation problem. In the method proposed here, simply a set of ordinary differential equations are to be solved. The whole method is quite efficient in computation. We have used the method to solve an MMIC (multimode interference coupler) problem as shown in Fig.1, where the input waveguide is single-mode with the core index 1.8 and the cladding index 1.446; and the multimode waveguide has the same refractive indices for the core and the cladding as the input waveguide. In the study here, we have assumed a TE wave is launched at the input end of the multimode waveguide. The electric field distribution at the output end of the multimode waveguide calculated by use of the proposed method with N=500 is shown in Fig.2 (a). To compare our method with others, we have also used a commercial software. Fig.2 (b) shows the corresponding electric field distribution obtained by BeamPROP with the approximation of Pade' order (4,4) (the output end of the multimode waveguide is at z=60un). It can be seen from Figs.2 (a) and (b) that the discrepancy between both results (i.e., in Figs. (a) and (b)) is quite negligible. Fig. 2(c) shows the corresponding result as Pade' (1,0) approximation is used by the commercial software. This is equivalent to the result obtained with paraxial approximation used, i.e., the result obtained with the term ${d^2E}/{dz^2}$ in Eq. (2) neglected.

III. Numerical results

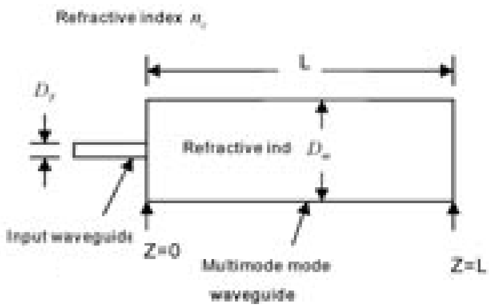

Fig.1 MMIC slab waveguide under study

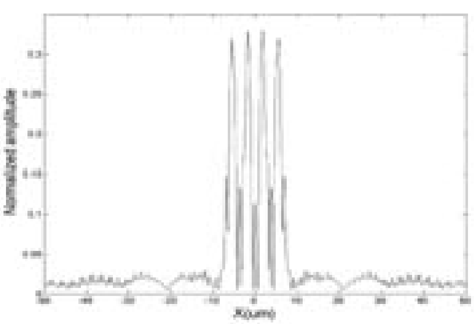

Fig.2(a)

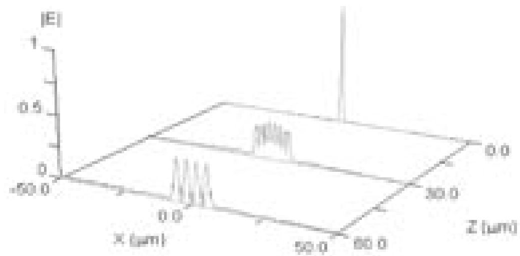

Fig. 2(b)

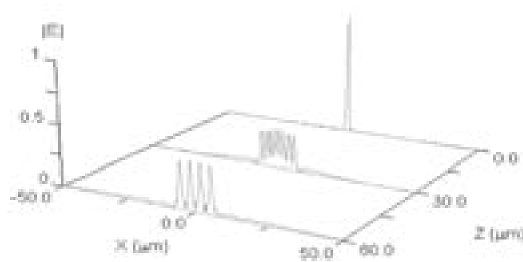

Fig. 2(c)

Fig,2 Electric field distributions of the MMIC shown in Fig. 2. (a) and (b) are the results obtained by use of the proposed method and BeamPROP, respectively. (c) is the result obtained with paraxial approximation used. The result of 2(a) is in good agreement with that of 2(b) at z=60um.

IV. Conclusion

In conclusion, we have proposed a new method for investigating the wave propagation in an optical waveguide with a longitudinally invariant structure. An explicit expression for the field at any longitudinal position along the waveguide can be obtained in this method, thus excluding the use of beam propagation algorithm for computation. The computational results are quite correct as they compared with a conventional BPM method. We strongly believe that the proposed method is quite efficient in computation.

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