

# A Multiobjective Evolutionary Solution for Short-Haul Airline Crew Pairing Problem

## 應用多目標演化計算於短程飛航組員之配對問題

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### Abstract

The Airline Crew Pairing Problem (ACPP) which consists of finding crew itineraries and satisfying the related law and regulation constraints is a significantly economic challenge. And many efforts have been spent by airline industry in the search for efficient and effective solutions. Instead of using the traditional set partitioning model, a different view is adopted here to model the crewing problem and formulate it with a set of combinational optimization equations.

In general, there are two phases in crew pairing, such as pairing generation and pairing optimization to be solved. A method of inequality-based multiobjective genetic algorithm (MMGA) is used here to provide the solution and solve them at the same time. Besides, with the Method of Inequalities (MOI), designers can configure the ranges

of solutions by adjusting an auxiliary vector of performance indices. In practice, the proposed MMGA approach possesses the merits of global exploration and can provide several optimal or feasible solutions to help planners perform efficient and effective decision-making.

Keywords: crew pairing, multiobjective genetic algorithms, combinational optimization

### 摘要

飛航組員之配對問題包含了搜尋組員的排程路線及配合相關的法律及規範限制。此一問題牽涉到相當大的飛航經濟成本，許多航空公司一直花費許多的人力、財力，尋求經濟及有效的解決方案。有別於傳統所使用的集合-分割方式，本論文採用不同的觀點建立配對模型並將此問題轉換成組合最佳化之問題。

一般而言，此配對問題牽涉到產生配對組合及配對最佳化兩個階段。論文中，將利用基於不等式之多目標遺傳演算法

(MMGA)同步討論並求解出此兩階段問題。此外，藉由不等式之操作，設計者可以設定效率指標輔助向量以調整相關解集合的範圍。在實務上，此一方法具備了全域的搜尋能力並能產出多個最佳或可行解，對於規劃者，將可作為一實際且有效的決策工具。

關鍵詞：飛航組員配對、多目標遺傳演算法、組合最佳化

## 1. Introduction

The airline scheduling which mainly contains the scheduling of aircraft maintenance, the routing for aircraft and the crew scheduling affects the most costs and benefits of the airline company. In the crew scheduling part, all flights which are assigned to the aircrafts according to the routing schedule require the personnel, such as pilots and crew members. Due to the laws and regulations, the working hours of personnel are limited. Therefore, the flights assigned to one aircraft should be separated to several sets so they can be assigned to several groups of crew members.

A pairing for crews is a sequence of flight duties, starting and ending at a crew base. An overnight connection between two duties is usually called layover and the airline company needs to pay extra cost for such conditions. Hence, the main goals of the crew pairing problem are shown as follows.

- To minimize number of groups
- To minimize layover number
- To satisfy the laws and regulations

In general, the crew pairing problem can be categorized as three types of problems

according to the periodical cycle, such as daily problem, monthly problem, and dated problem (Gopalakrishnan and Johnson, 2005). The schedules form a cycle in one day, one week, and one month, respectively. In this paper, the focus is to deal with the daily case.

From the solution steps, there are two main phases, pairing generation and pairing optimization, to provide a solution for the daily crew pairing. Most researches use enumeration way in the former phase. The drawbacks of enumeration are the solution space will be limited and time consuming for planners. Therefore, we use the genetic algorithms (GA) to integrate both phases. Genetic algorithms, first introduced by Holland, were later improved by many researchers (Holland, 1975; Leung and Wang, 2001; Deb, 2003; Tsai et al., 2004). GAs possess the global explorer capabilities and have been successfully used in many multi-objective researches (Lee et al., 2007, Chou, 2008).

In this paper, the airline crew pairing problem would be formulated into combination optimization equations and the optimal or feasible solutions would be globally searched by using a method of inequality-based multiobjective genetic algorithm (MMGA). A real-world case study would be presented later to show the good pairing capabilities of the proposed approach.

## 2. Related Works

A detailed survey of aircrew pairing

problems can refer Gopalakrishnan and Johnson (2005). And there have been more researches on crew scheduling. For example, Arabeyre (1969) surveyed older work on crew scheduling. Etschmaier and Mathaisel (1985) provided a more recent survey. Some more recent algorithms and practices have been proposed as the column generation approach to solve the crew pairing problem (Crainic and Rousseau, 1987; Lavoie et al., 1988; Hoffman and Padberg, 1993).

### 3. Mathematical Models

In this section, the mathematical models are described first and then, the objective functions and the definition of auxiliary performance index vector is described later.

#### Notations

$\alpha$ : number of group of crew members

$\beta$ : maximal number of daily flights assigned to each group of crewmembers

$\gamma$ : number of flights

$\mu$ : number of possible pairings suggested by planners

$f_i$ : identifier of the  $i^{\text{th}}$  flight,  $1 \leq i \leq \gamma$ , and the set of  $F$  is denoted as  $F = \{1 \leq i \leq \gamma\}$ .

Also, various associated information of each  $f_i$  are listed as follows.

$f_i$ : identifier of  $f_i$ ,

$\hat{p}_i$ : origin of  $f_i$ ,

$\bar{p}_i$ : destination of  $f_i$ ,

$\hat{t}_i$ : departure time from  $\hat{p}_i$

$\bar{t}_i$ : arrival time in  $\bar{p}_i$ ,

To overcome this time-consuming problem, an improved form of candidate

solutions is proposed as:

$$\mathbf{S} = \{s_{i,j} | s_{i,j} \in F \cup \{-1\}\} \quad (1)$$

where  $S$  is a two-dimensional matrix of  $\alpha \times \beta$  elements, and each  $s_{i,j}$  represents a flight identifier which means the  $j^{\text{th}}$  flight assigned to the  $i^{\text{th}}$  group of crew member. To keep the number of flights assigned to each group identical, we assign dummy flights with flight identifier -1.

The main feature of the proposed model is that the number of pairings becomes to a controllable variable instead of unexpected value within the range  $0 \leq \mu \leq 2^\beta - 1$ . This is useful when performing practical pairing process since the number of pairing is related to the manpower in the airline company.

The goal of aircrew pairing problem is to make the total cost to be minimized. Therefore, the objective functions to be minimized, such as ground turn-around time, crew connection, number of layover, and flight duty period are described as follows.

Ground turn-around time objective ensures that each aircraft has sufficient ground turn-around time not less than the legal ground turn-around time, denoted as  $T_X$ , to be allowed for the subsequent flight. The objective is defined as

$$\phi_1(\mathbf{S}) = \sum_{i=1}^{\alpha} \sum_{j=1}^{\beta-1} x_{i,j}^{(1)} \quad (2)$$

where

$$x_{i,j}^{(1)} = \begin{cases} 0 & \text{if } (\hat{t}_{i,j+1} - \bar{t}_{i,j}) \geq T_X \\ T_X - (\hat{t}_{i,j+1} - \bar{t}_{i,j}) & \text{otherwise} \end{cases}$$

Crew connection objective ensures that

the arrival airport of  $s_{i,j}$  is the same with the departure airport of  $s_{i,j+1}$  for each aircraft in  $\mathbf{S}$ , for  $1 \leq t \leq \alpha$ , and  $1 \leq j \leq \beta - 1$ . This objective is to reduce the extra cost of the nonprofit flight from  $\bar{p}_{i,j}$  to  $\hat{p}_{i,j+1}$ . The objective is defined as

$$\phi_2(\mathbf{S}) = \sum_{i=1}^{\alpha} \sum_{j=1}^{\beta-1} x_{i,j}^{(2)} \quad (3)$$

$$\text{where } x_{i,j}^{(2)} = \begin{cases} 0 & \text{if } \bar{p}_{i,j} = \hat{p}_{i,j+1} \\ 1 & \text{otherwise} \end{cases}$$

Layover objective ensures each group of crewmembers can start from and end to their home bases. Suppose the first and last flights of the  $i^{\text{th}}$  group in  $\mathbf{S}$  are  $s_{i,1}$  and  $s_{i,\text{last}}$ , respectively. The objective can be defined as

$$\phi_3(\mathbf{S}) = \sum_{i=1}^{\alpha} \kappa_i \quad (4)$$

$$\text{where } \kappa_i = \begin{cases} 0 & \hat{p}_{i,1} = \bar{p}_{i,\text{last}} \\ 1 & \text{otherwise} \end{cases}$$

According to the laws and regulations, the duty time of each aircrew pair should not be more than a legal time  $T_{FDP}$ . Therefore, the fourth evaluation function can be defined as follows.

$$\phi_4(\mathbf{S}) = \sum_{i=1}^{\alpha} \eta_i \quad (5)$$

$$\text{where } \eta_i = \begin{cases} 0 & \bar{t}_{i,\text{last}} - \hat{t}_{i,1} \leq T_{FDP} \\ 1 & \text{otherwise} \end{cases}$$

In other words, if the total flight duty time of one aircrew pair exceeds the legal time  $T_{FDP}$ , the evaluation function  $\phi_4(\mathbf{S})$  will be added the excessive time, or the

violation time.

### Definition of Auxiliary Performance Index Vector

In original formulations of multiobjective optimization, the set of admissible bounds are not considered. To make the admissible bounds be considered in multiobjective optimization, the auxiliary performance index is proposed. The original objectives are transformed into the auxiliary performance index vector:

$$\Lambda(\mathbf{S}, \boldsymbol{\varepsilon}) = (\lambda_1(\mathbf{S}, \varepsilon_1), \lambda_2(\mathbf{S}, \varepsilon_2), \lambda_3(\mathbf{S}, \varepsilon_3), \lambda_4(\mathbf{S}, \varepsilon_4), \lambda_5(\mathbf{S}, \varepsilon_5)) \quad (6)$$

$$\text{where } \lambda_i(\mathbf{S}, \varepsilon_i) = \begin{cases} 0 & \text{if } \phi_i(\mathbf{S}) \leq \varepsilon_i \\ \phi_i(\mathbf{S}) - \varepsilon_i & \text{otherwise.} \end{cases}$$

The auxiliary performance index vector related to the inequalities is converted from the MOI problem to a multiobjective optimization problem. The multiobjective formulation using the auxiliary performance index vector is useful for MOI since the admissible bounds can be combined to all objectives. Therefore, each objective can be transformed to the form of inequalities.

### Formulation of the Aircraft Routing Problem

As mentioned above, this problem comprises of multiple small-the-best objectives. Instead of combining these objectives into a single scalar, the aircraft routing problem with multiple objectives can be formulated as follows.

$$\text{Minimize } \lambda_i(\mathbf{S}, \varepsilon_i), \quad 1 \leq i \leq 4 \quad (7)$$

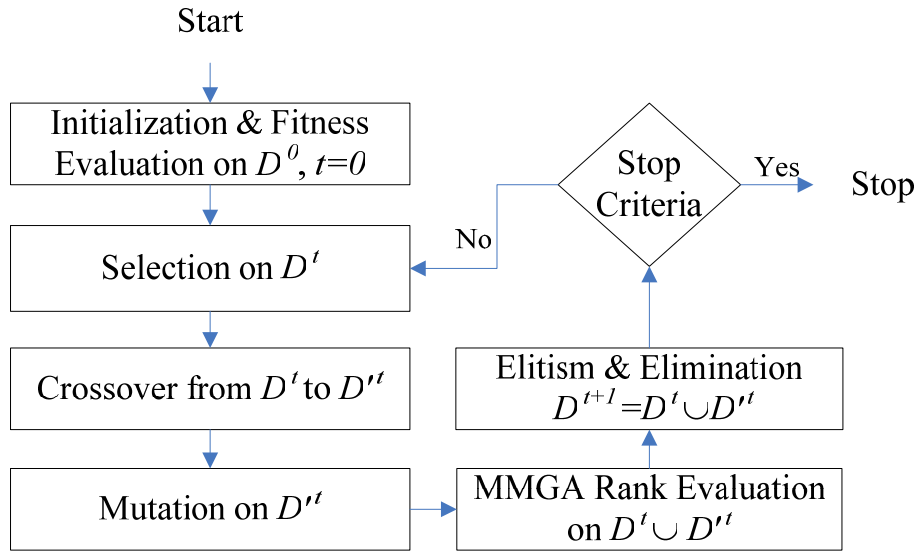
$$\text{subject to } S = [s_{i,j}]_{\alpha \times \beta}$$

#### 4. Solution by Using MMGA

For a method of inequalities (MOI), MMGA employs the global search capability of genetic algorithms and proposes an auxiliary vector performance index which would be related to the set of design specifications and can be multi-objective optimized according to the assigned fitness based on Pareto-ranking rules. An applied auxiliary vector index can always generate tunable parameters belonging to a strictly Pareto optimal set and provide the planners useful information for adjusting the design specifications. A

heuristic Pareto algorithm was also provided to lower the Pareto computation costs. A diversity consideration on the population was invited into the algorithm to avoid the effect of the generic drift (bias) and premature convergence.

The flow chart of the algorithm can be summarized in Figure 1. Just like the general multi-objective genetic algorithm (MOGA), evolutionary population should be operated by iterations through initialization, fitness computation, multiobjective evaluation, crossover to generate offspring, mutation and selection for elimination.



$D^t$ : Population set in generation  
 $D''$ : Off-spring set in generation

Figure 1. Flow chart diagram

And the detailed algorithm is described as the follows.

##### MOI-Based Multiobjective GA (MMGA)

- Input:**
- (1) A set of candidate solutions  $\mathbf{D}^{(t)} = \{\mathbf{S}_1^{(t)}, \mathbf{S}_2^{(t)}, \dots, \mathbf{S}_n^{(t)}\}$  with population  $n$  in generation  $t$ .
  - (2) Two temporary sets of

candidate solutions:  $\mathbf{D}^{(t)}$ ,  $\mathbf{E}^{(t)}$ .

- (3) The admissible bound vector  $\boldsymbol{\varepsilon}$ .

**Output:** A set of optimal candidate solutions within meeting the requirements of admissible bounds.

- Step 1:** Determine the MMGA parameters: population size  $n$ , maximum number of generations  $g$ , crossover rate  $r \in [0, 1]$ , and mutation rate  $\mu \in [0, 1]$ .
- Step 2:** Determine the admissible bound vector  $\varepsilon = \{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4\}$  of the transition time objective, passenger crew objective, layover objective, flight duty period objective, and working hour objective.
- Step 3:** Let  $t:=0$ . Initialize the population  $\mathbf{D}^{(t)}$ .
- Step 4:** Adopt the repairing process to adjust all chromosomes for the violation of time constraint violation.
- Step 5:** Evaluate the auxiliary performance index vector of each individual  $\mathbf{S}^{(t)}$  in entire population  $n$ .
- Step 6:** Apply improved rank-based fitness assignment method to calculate the fitness of each individual  $\mathbf{S}^{(t)}$ .
- Step 7:** If the number of current generation  $t$  reaches  $g$ , or all the objectives are satisfied, then stop the algorithm.
- Step 8:** Choose two individuals using the tournament selection method.
- Step 9:** Perform crossover and mutation operators to generate the populations of next generation  $t+1$  in the mating pool  $\mathbf{D}^{(t)}$ . The mutation operator randomly selects two flights in the chromosome and exchanges their positions.
- Step 10:** Adopt the repairing process for the chromosomes in  $\mathbf{D}^{(t)}$ .
- Step 11:** Evaluate the auxiliary performance index vector of each individual in  $\mathbf{D}^{(t)}$ .
- Step 12:**  $\mathbf{D}^{(t)} = \mathbf{D}^{(t)} \cup \mathbf{D}^{(t)}$ .
- Step 13:** Adopt improved rank-based fitness assignment method again to calculate the fitness of each individual in  $\mathbf{D}^{(t)}$ , and let  $t:=t+1$ . Go to Step 7.

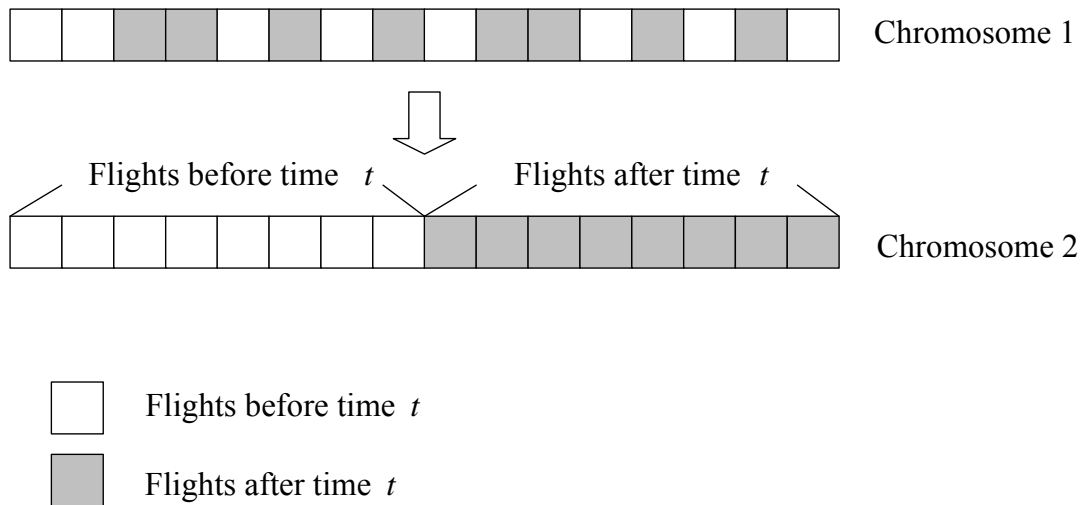


Figure 2. The representation of one individual

## Encoding Scheme

The encoding scheme of each individual is a two-dimensional matrix. To make the encoding more efficiency, we transform the chromosome to a string. To satisfy the objective of working hour, we use a modified approach to reduce the complexity on solving the working hour objective. In each individual, the flights that are earlier than time  $t$  are allocated in the left-hand side of the individual. On the other aspect, the flights that are later than time  $t$  are put in the right-hand-side of the individual as Figure 2.

## Selection Operation

The roulette-wheel selection is adopted to select the best fitting individuals of the population into a mating pool. The selection probability  $B$  of individual is defined as follow:

$$B(i) = \frac{F(i)}{\sum_{i=1}^n F(i)} \quad (8)$$

where  $F(i)$  is the fitness of the individual  $i$ , and  $n$  is the population size.

## Crossover

In the crossover process, we use an order-based crossover. First, a random mask is generated to determine which flights are fixed, and flights are to be changed. If the  $i^{\text{th}}$  element of the generated mask is 1, then the  $i^{\text{th}}$  gene of offspring1 is fixed. Otherwise, it will be replaced. As shown in Figure 3, the fixed genes  $\{1, 3, 4, 7, 9, 11, 12, 14\}$  on both offspring1 and offspring2 will be kept in the original positions. According to Figure 3, the genes to be replaced on each offspring are in the following order:

Offspring1:  $2 \rightarrow 5 \rightarrow 6 \rightarrow 8 \rightarrow 10 \rightarrow 13$

Offspring2:  $8 \rightarrow 5 \rightarrow 2 \rightarrow 6 \rightarrow 10 \rightarrow 13$

After the process of crossover, the orders of the genes are exchanged according to the following order:

Offspring1:  $8 \rightarrow 5 \rightarrow 2 \rightarrow 6 \rightarrow 10 \rightarrow 13$

Offspring2:  $2 \rightarrow 5 \rightarrow 6 \rightarrow 8 \rightarrow 10 \rightarrow 13$

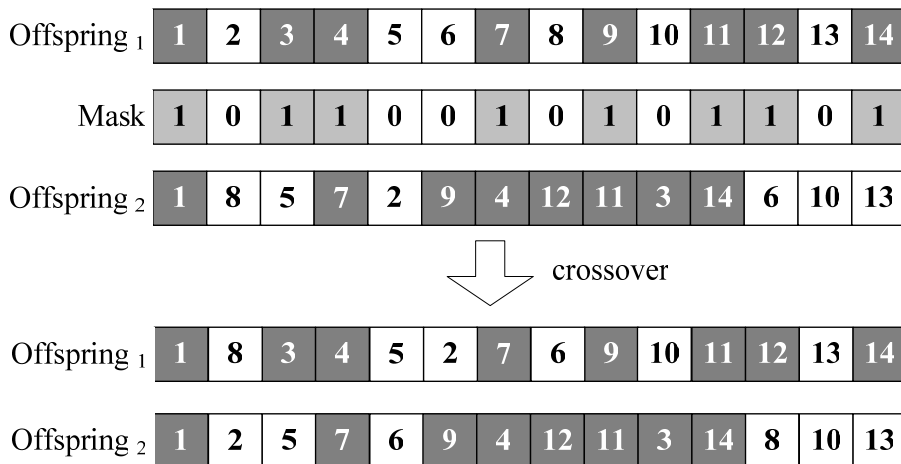


Figure 3. Order-based crossover

## Mutation

Also, we use an improved mutation

operator as the Figure 4. The individual are temporarily transformed to the conceptual

model of 2-dimensional matrix, i.e. each row is the set of flights assigned to a group of crewmembers.

When selecting the genes to be exchanged, only the segments with

violations have more chances to be selected. This can prevent extra costs of inefficient search.

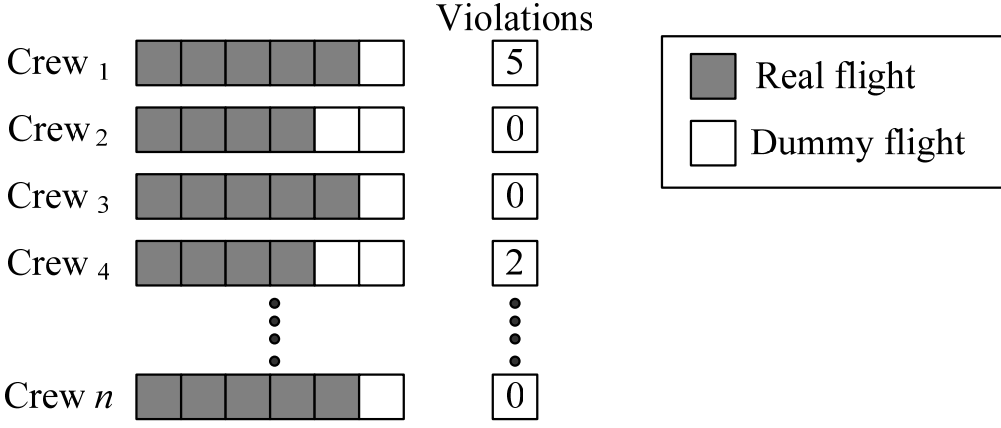


Figure 4. Mutation operation

### 5. Experiments

In this section, we demonstrate the experimental results. The experiment focuses on the practical timetable of MD90. There are 70 flights in the aircraft routing for pairing crews. The goal is to find out a crew pairing schedule that matches all objectives. Ideally, each pairing should have

the same origin and destination without layover costs.

As shown in Figure 5, the pairing case from the solution set is feasible in all objectives. The duty periods of all groups are within 10 hours. And Figure 6 shows the convergence diagram about the various objective values of the top chromosome.

### 6. Conclusion

The goal of this research is to solve the complex pairing problem by using MMGA approach and to demonstrate that this method is capable of reducing solution time which is verified in the real world. Results obtained from the case of a short-haul domestic airline in Taiwan shows clearly the advantages of solving the pairing problem.

With the global explorer capabilities of GAs, the pairing generation and pairing optimization can be solved at the same time.

The experiment results for MD90 show the good pairing solutions which optimize various objectives such as crew turn around time, crew connection, Layover time and flight duty period.



	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24				
C1			803 TSA KHH		810 KHH TSA		811 TSA KHH		818 KHH TSA														
C2				805 TSA KHH		812 KHH TSA		813 TSA KHH		820 KHH TSA													
C3			806 KHH TSA		807 TSA KHH		814 KHH TSA		815 TSA KHH														
C4	804 KHH TSA	881 TSA TNN		884 TSA TNN	883 TSA TNN		886 TSA TNN		887 TSA TNN														
C5	851 TSA TTT	852 TTT TSA				681 TSA KNH		682 KNH TSA		683 TSA KNH		684 KNH TSA											
C6		853 TSA TTT		854 TTT TSA		855 TSA TTT		856 TTT TSA		857 TSA TTT		858 TTT TSA											
C7	902 KHH TPE			901 TPE KHH		659 KHH MZG		660 MZG KHH															
C8		882 TSA TNN		603 TSA MZG		602 MZG TSA		605 TSA MZG		606 MZG TSA		889 TSA TNN											
C9												817 TSA KHH		824 KHH TSA		823 TSA KHH		830 KHH TSA		833 TSA KHH		840 KHH TSA	
C10													819 TSA KHH		826 KHH TSA		825 TSA KHH		832 KHH TSA				
C11													822 KHH TSA		821 TSA KHH		828 KHH TSA		829 TSA KHH		836 KHH TSA		841 TSA KHH
C12																							
C13																							
C14																							
C15																							

Figure 5. Crew pairing of 15 groups

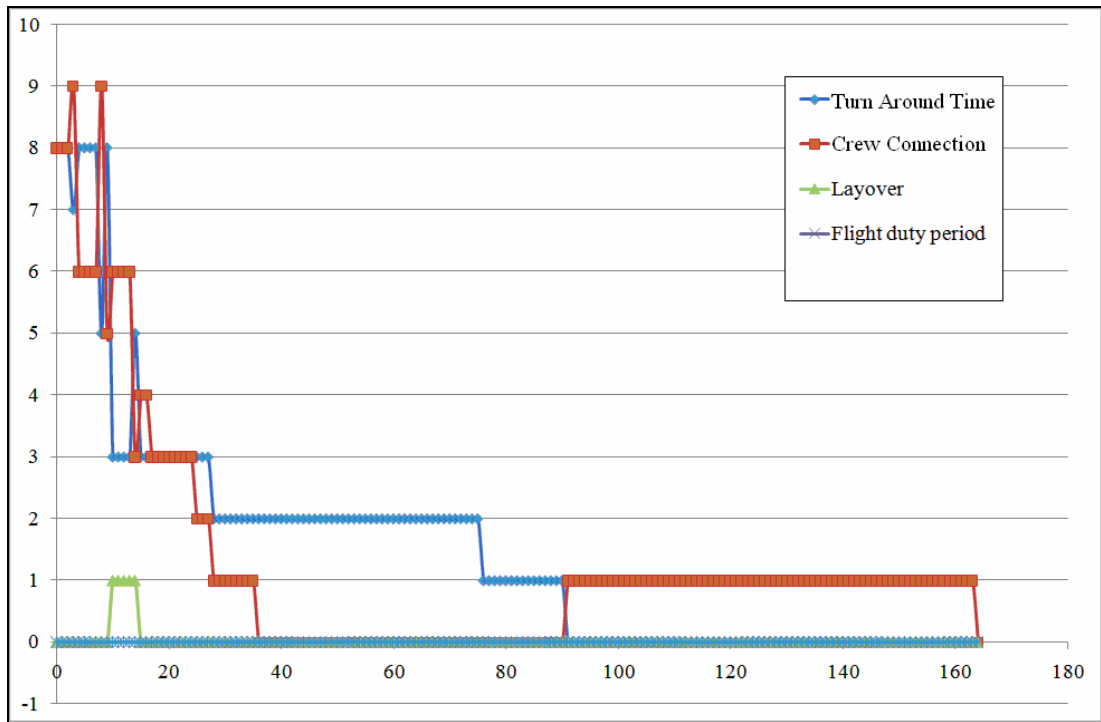


Figure 6. Convergence of MD90 schedule

## 7. Acknowledgement

This work was partially supported by the National Science Council, Taiwan, Republic of China, under grant numbers NSC 95-2516-S-327-001, NSC 96-2221-E-327-027 and NSC 96-2221-E-327-005-MY.

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